Lecture 9

2020/2021

Microwave Devices and Circuits for Radiocommunications

2020/2021

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **associate professor Radu Damian**
  - Wednesday 15-17, Online, Microsoft Teams
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - **all materials/equipments authorized**

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, “Microwave Engineering”**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/online exam
  - used at lectures/laboratory

Profile photo

- Profile photo – online “exam”

Examene online: 2020/2021

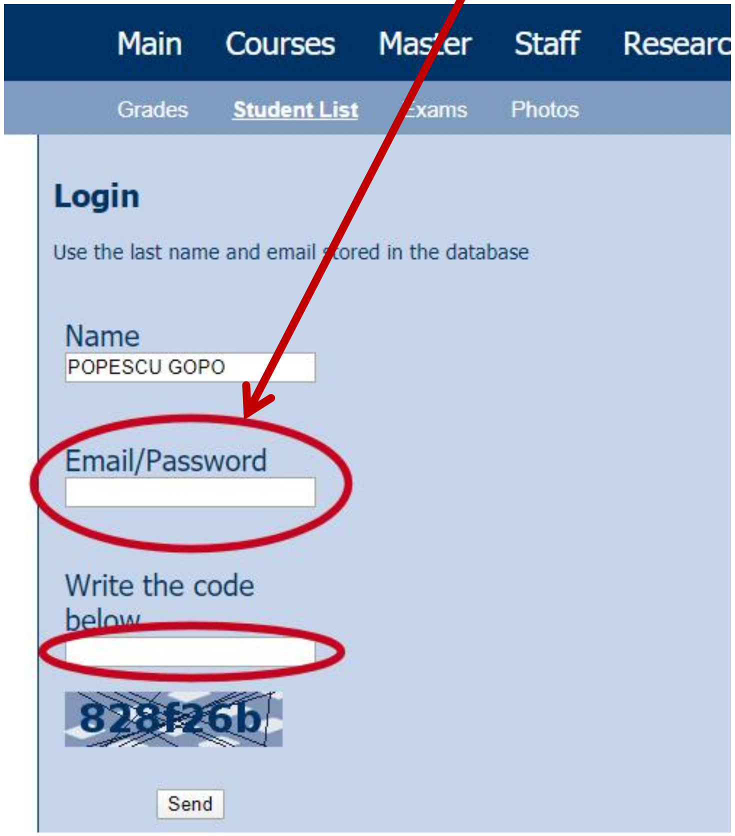
Disciplina: MDC (Microwave Devices and Circuits (Engleza))

Pas 3

Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..

Online

- access to **online exams** requires the **password** received by email



Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

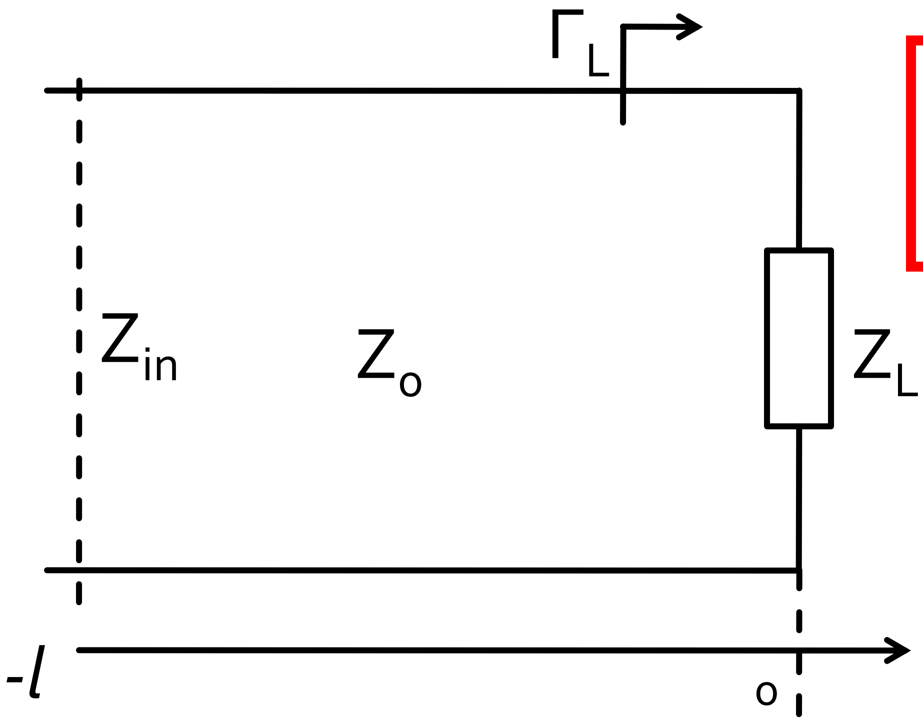
Grade = Quality of the work +

+ Quality of the submission

Important

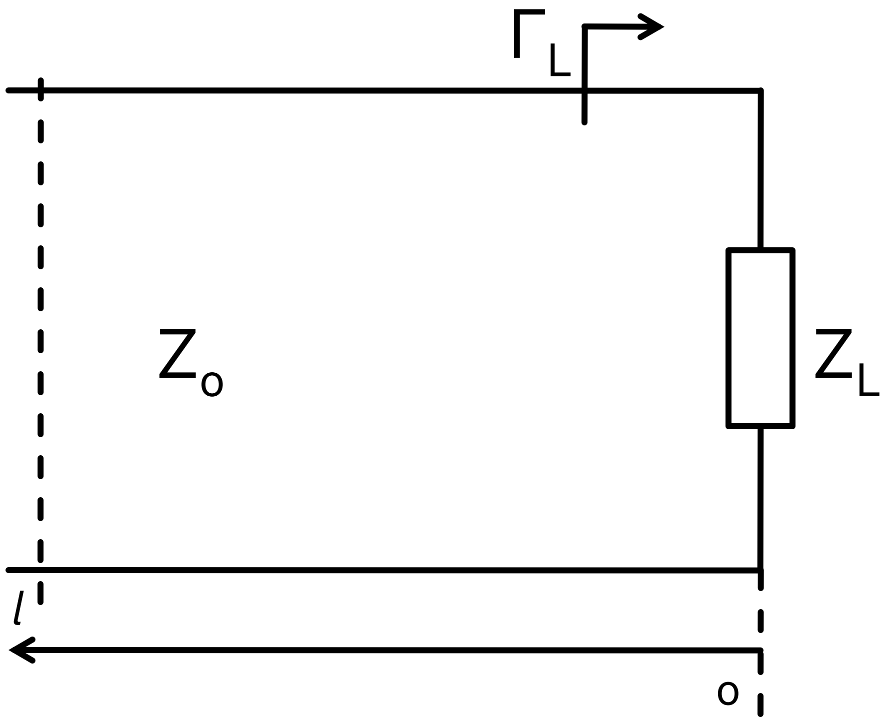
The lossless line

- input impedance of a length $\boldsymbol{l}$ of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

- Z_o real

The lossless line

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z)=\frac{V_0^+}{Z_0}\cdot\left(e^{-j\cdot\beta\cdot z}-\Gamma\cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

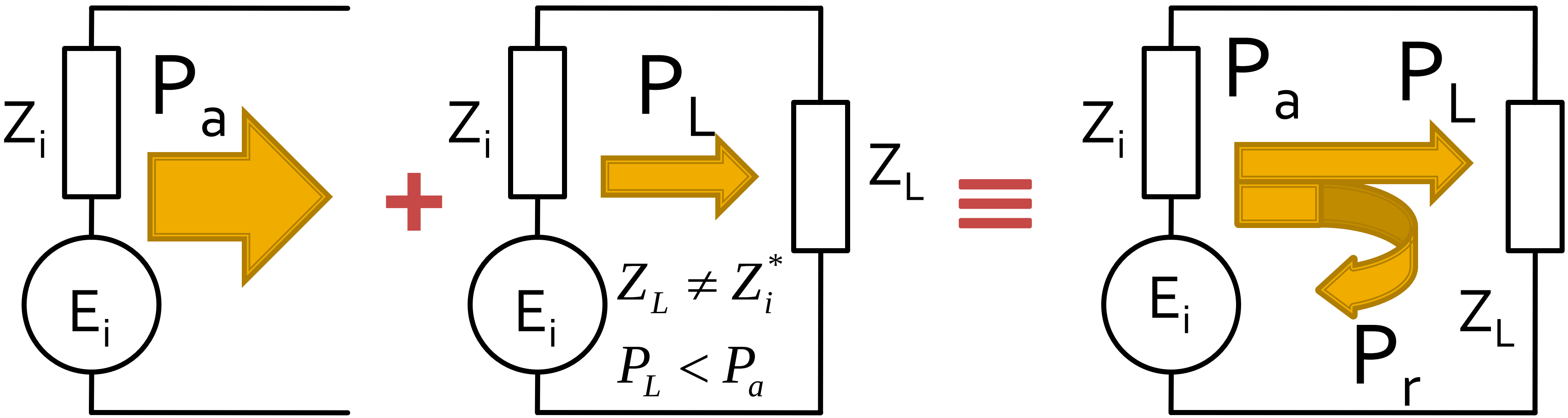
$$P_{avg}=\frac{1}{2}\cdot\mathrm{Re}\left\{V(z)\cdot I(z)^*\right\}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\mathrm{Re}\left\{1-\Gamma^*\cdot e^{-2j\cdot\beta\cdot z}+\Gamma\cdot e^{2j\cdot\beta\cdot z}-\left|\Gamma\right|^2\right\}$$

$$\left(z-z^*\right)=\mathrm{Im}$$

$$P_{avg}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\left(1-\left|\Gamma\right|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] $\qquad \mathrm{RL}=-20\cdot\log\left|\Gamma\right| \quad [\mathrm{dB}]$

Reflection and power / Model



- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

Matching , from the point of view of power transmission

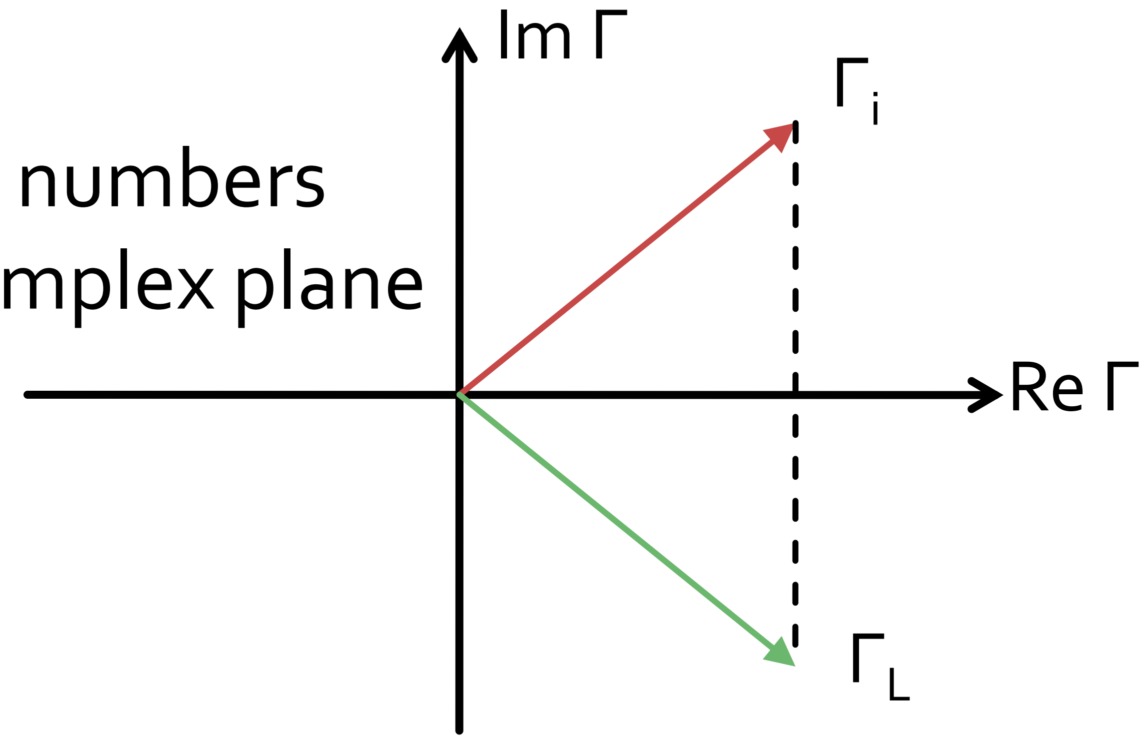
$$\boxed{Z_L = Z_i^*}$$

If we choose a real Zo

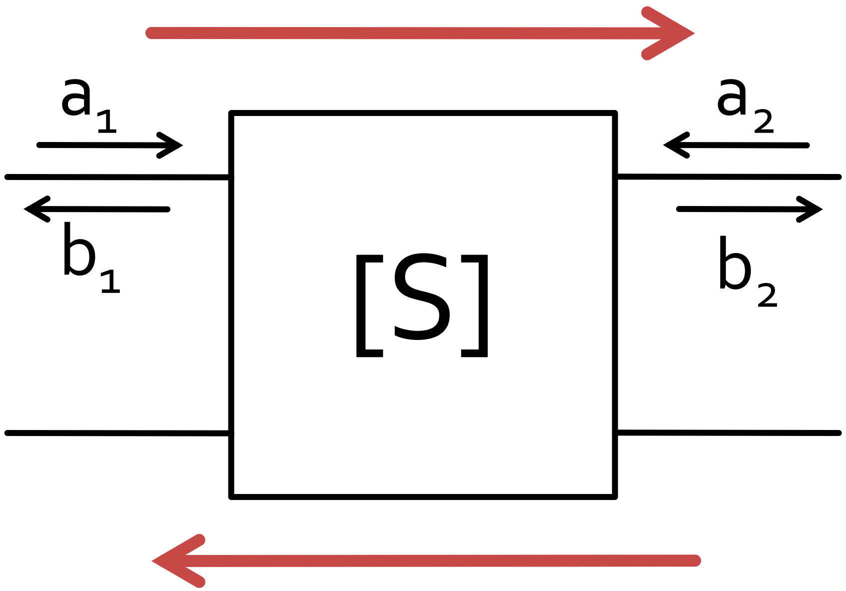
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\boxed{\Gamma_L = \Gamma_i^*}$$

- complex numbers
- in the complex plane



Scattering matrix – S

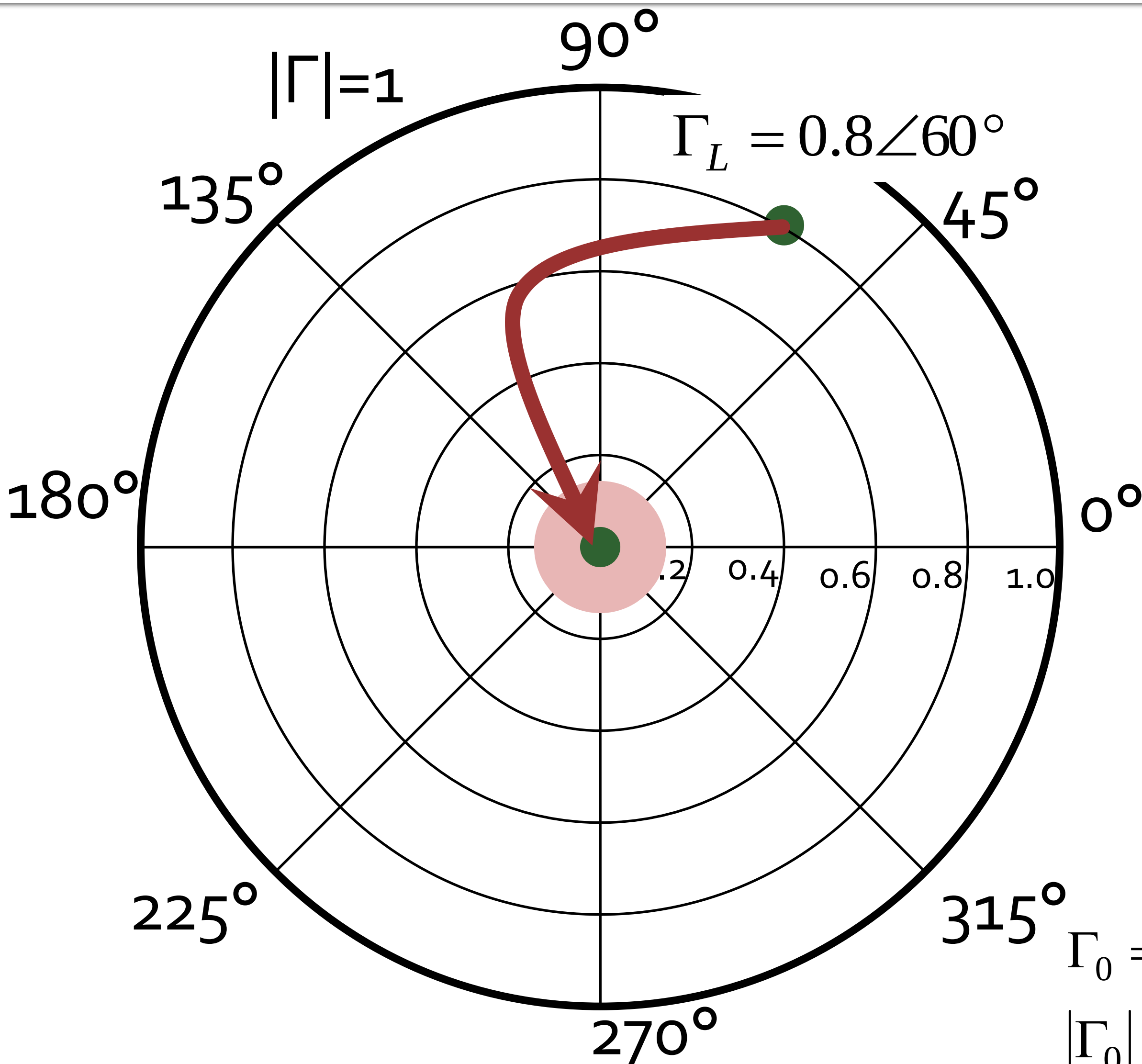


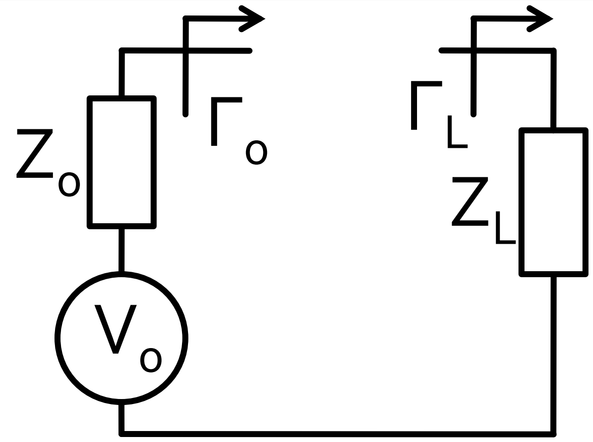
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Impedance matching



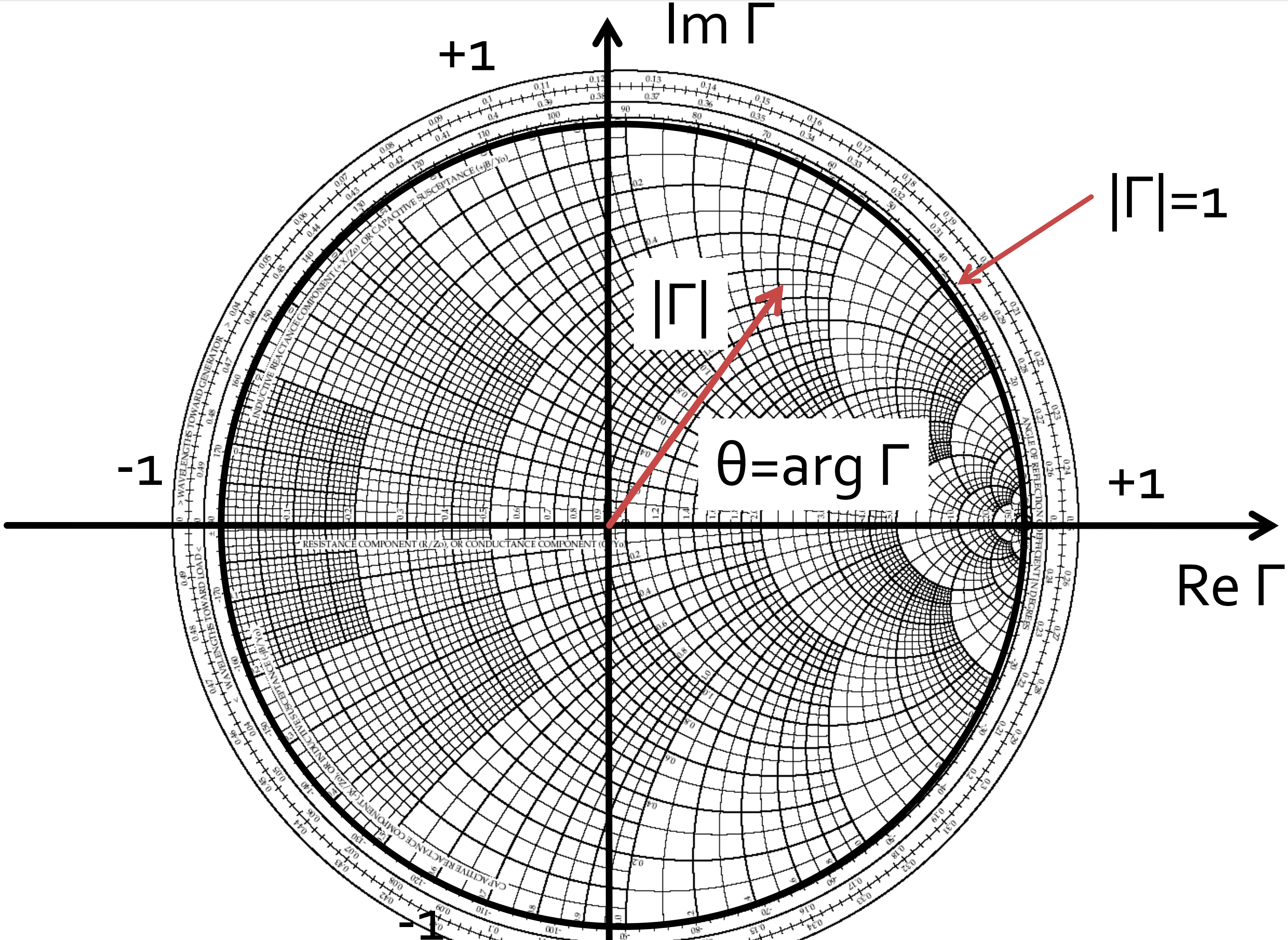


How?

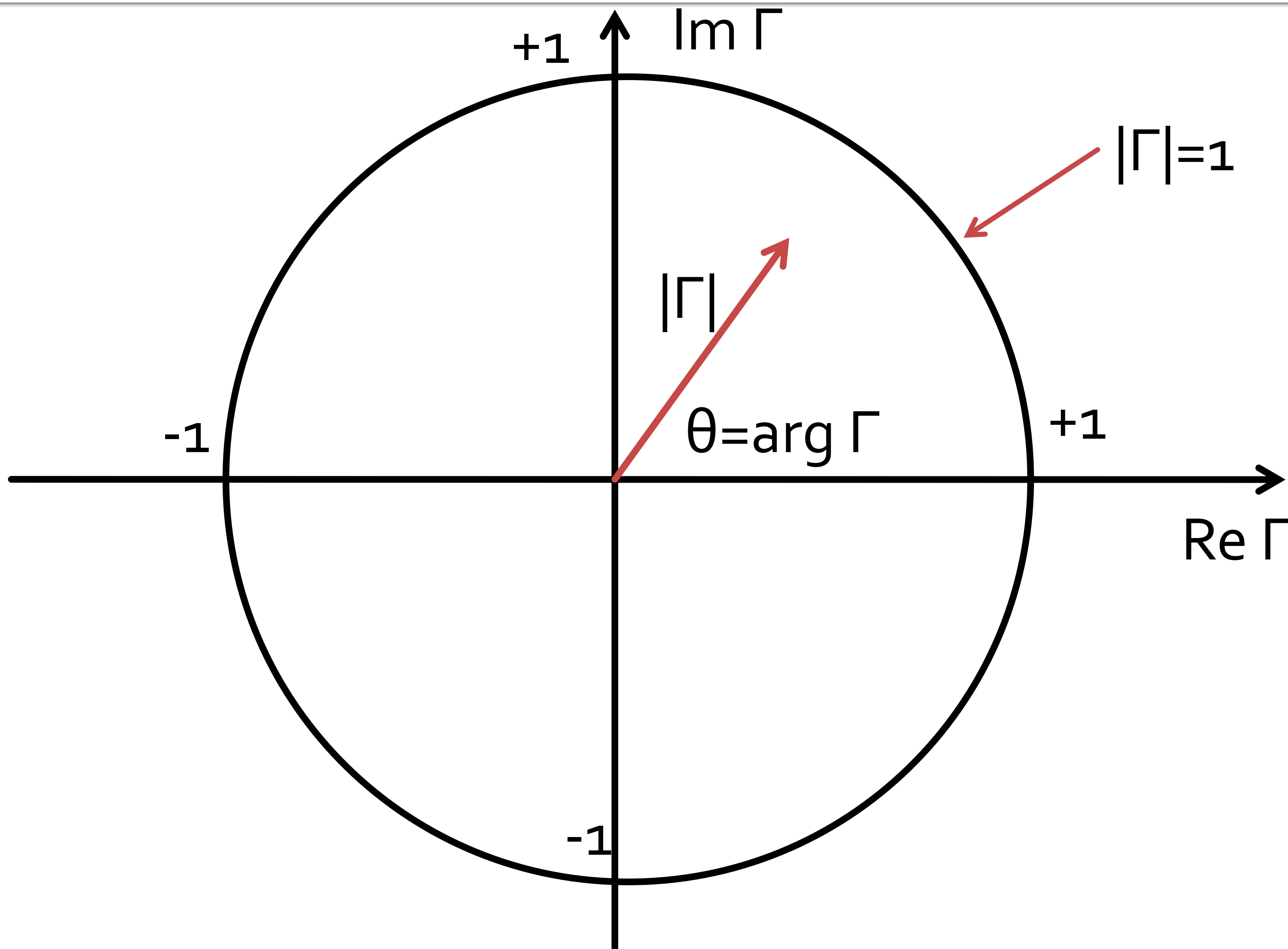
$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \le \Gamma_m$ "good enough" match

The Smith Chart



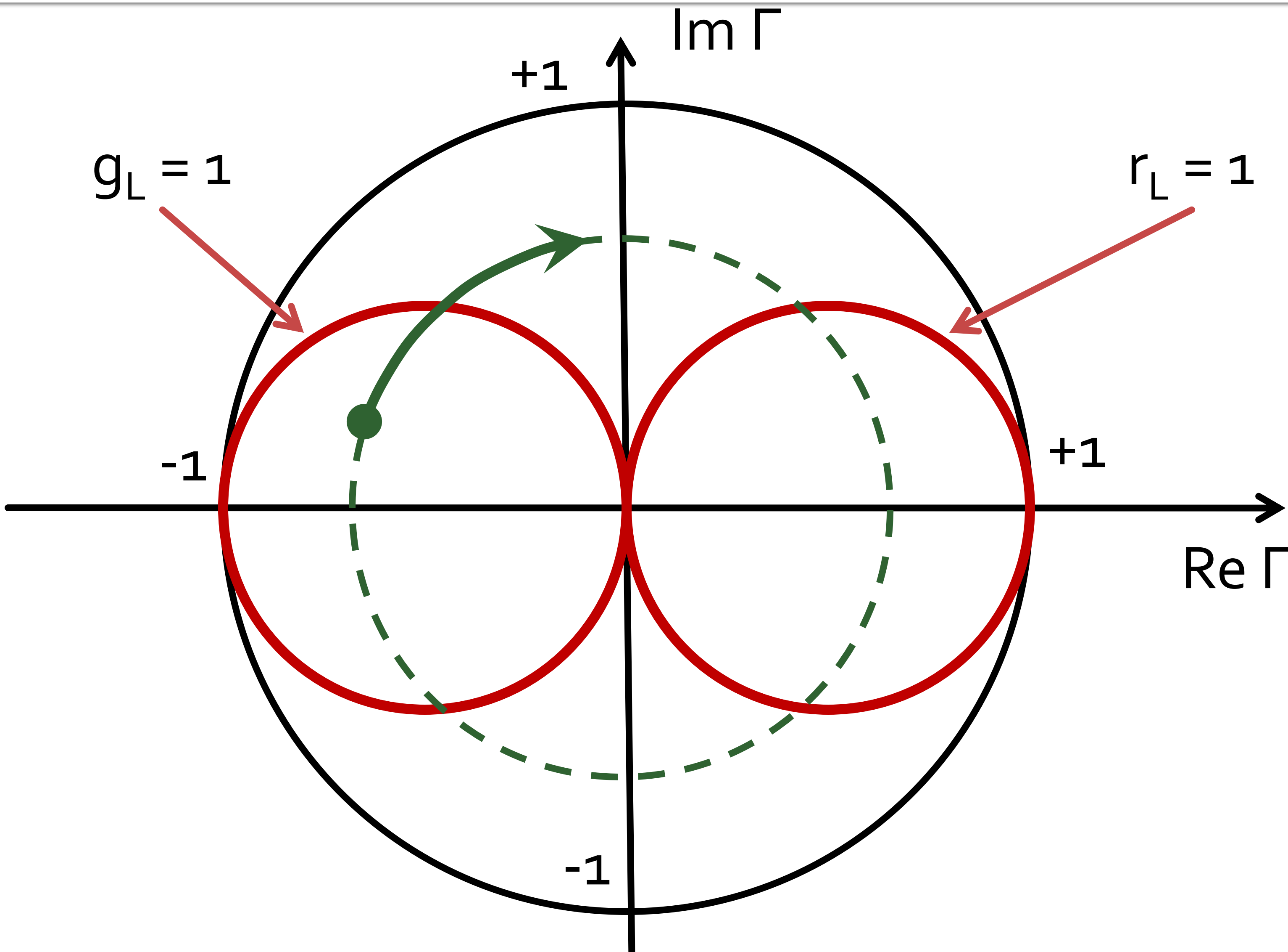
The Smith Chart



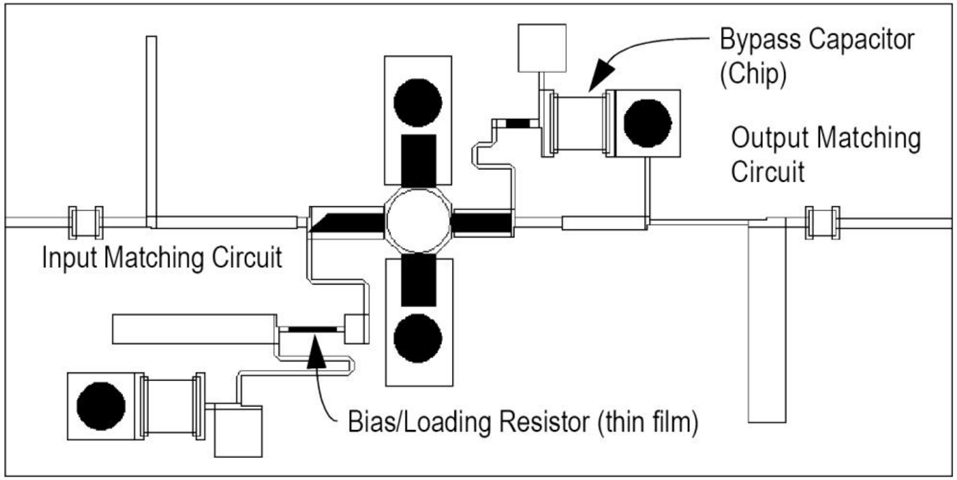
Impedance Matching

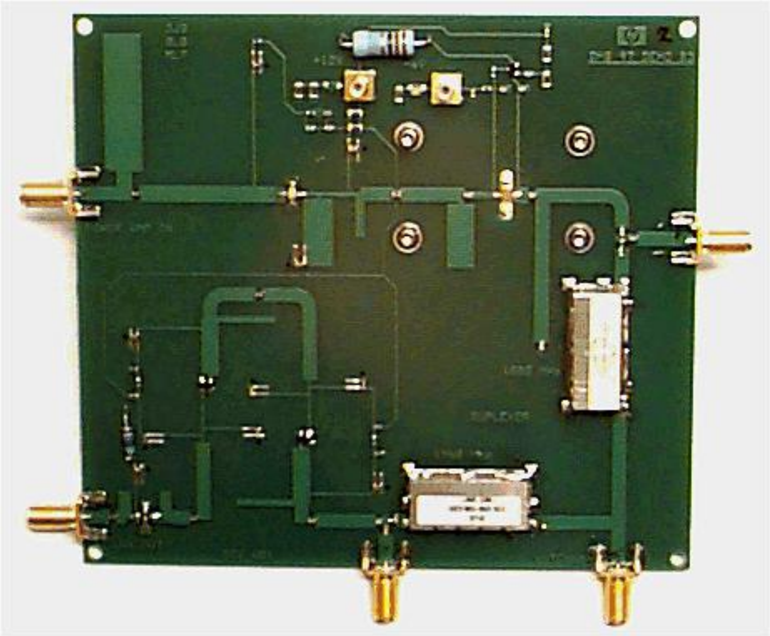
Impedance Matching with Stubs

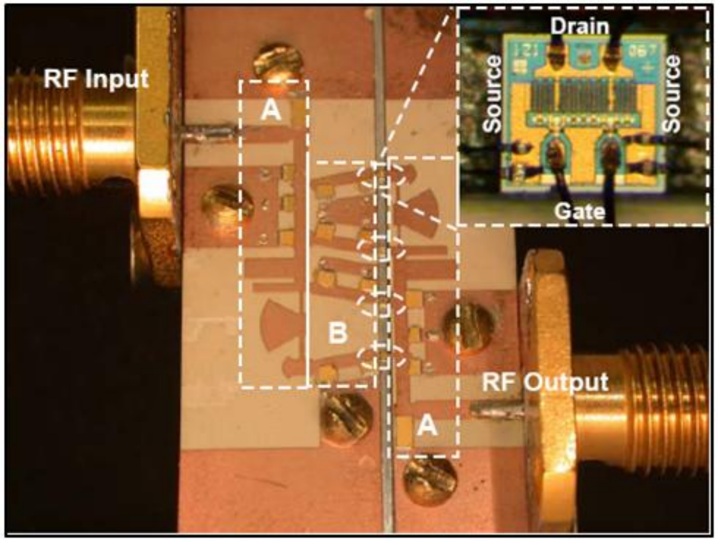
Smith chart, r=1 and g=1

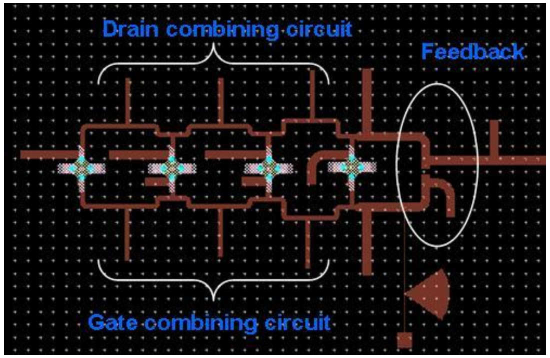


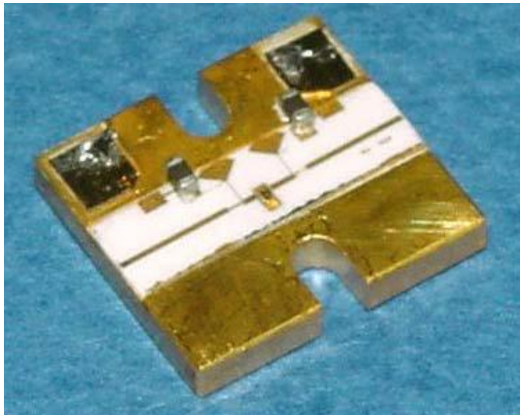
Impedance Matching with Stubs









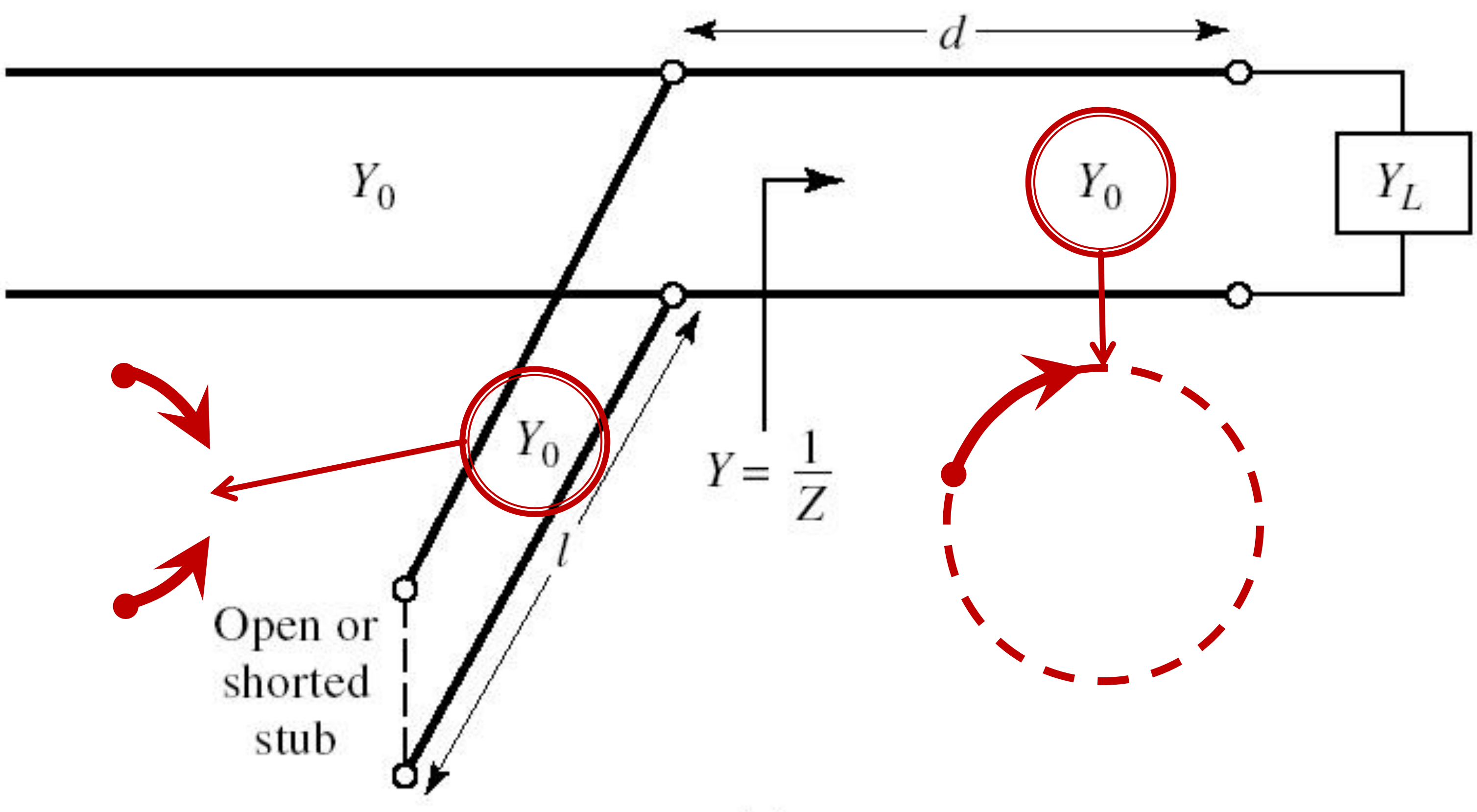


Analytical solutions

Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Analytical solution, usage

$$\boxed{\cos(\varphi + 2\theta) = -|\Gamma_S|}$$

$$\boxed{\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}}$$

$$\Gamma_S = 0.593 \angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

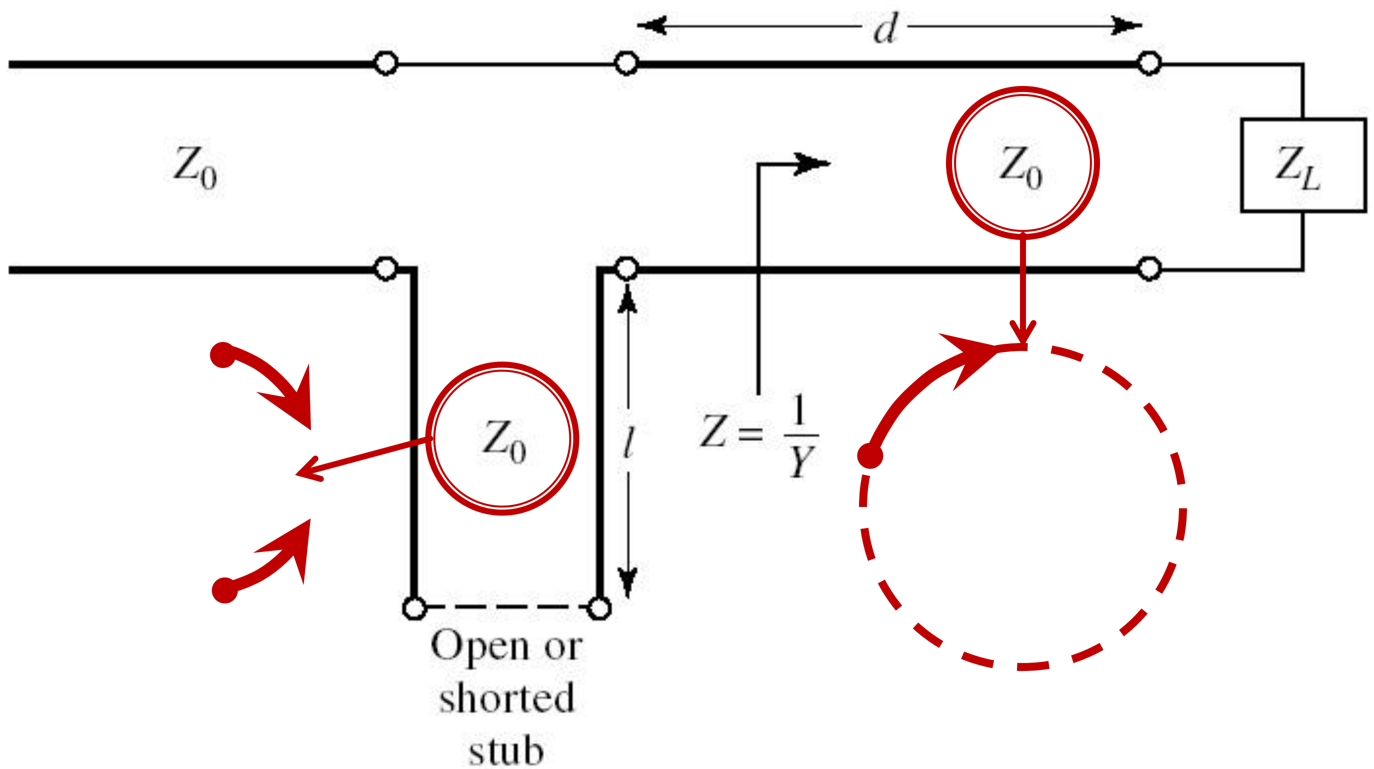
    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$
  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Analytical solution, usage

$$\boxed{\cos(\varphi + 2\theta) = |\Gamma_S|}$$

$$\boxed{\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}}$$

$$\Gamma_S = 0.555 \angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \mathrm{Im}\, z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\mathrm{Im}\, z_S) = -36.8°(+180°) \rightarrow \theta_{ss} = 143.2°$$

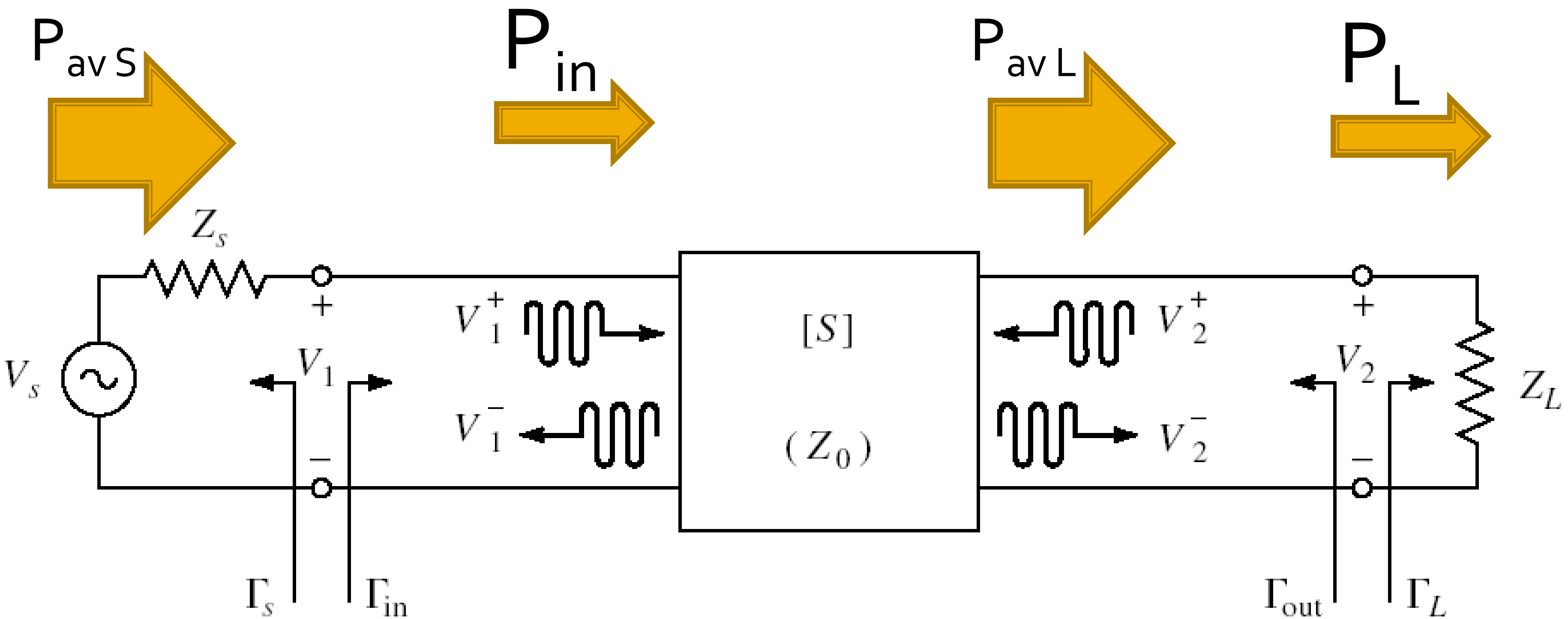
  - **"-" solution**

$$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2°(+180°) \rightarrow \theta = 166.8°$$

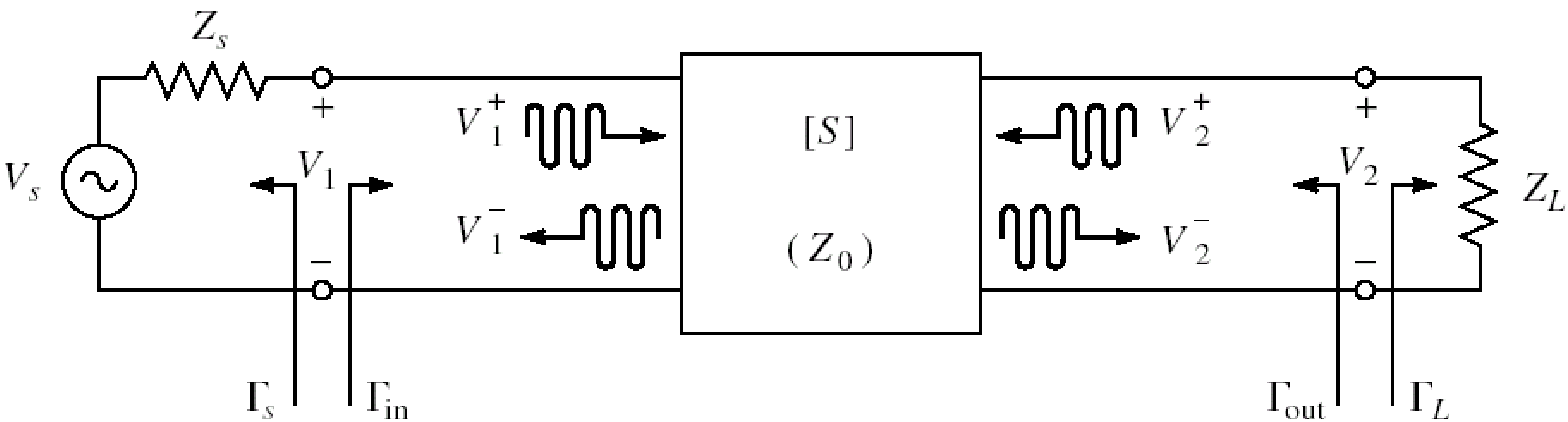
$$\mathrm{Im}\, z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\mathrm{Im}\, z_S) = 36.8°$$

Amplifier Power / Matching

- Two ports in which matching influences the power transfer



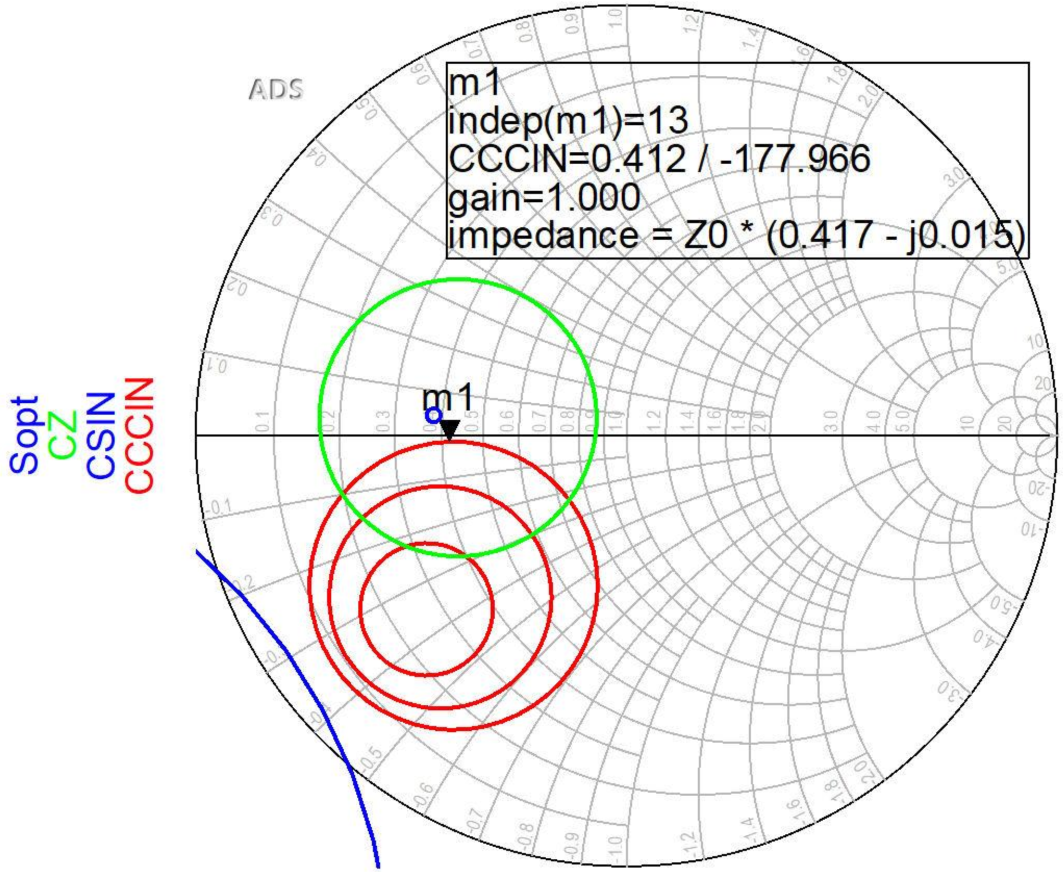
Amplifier as two-port



$$\boxed{\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}}$$

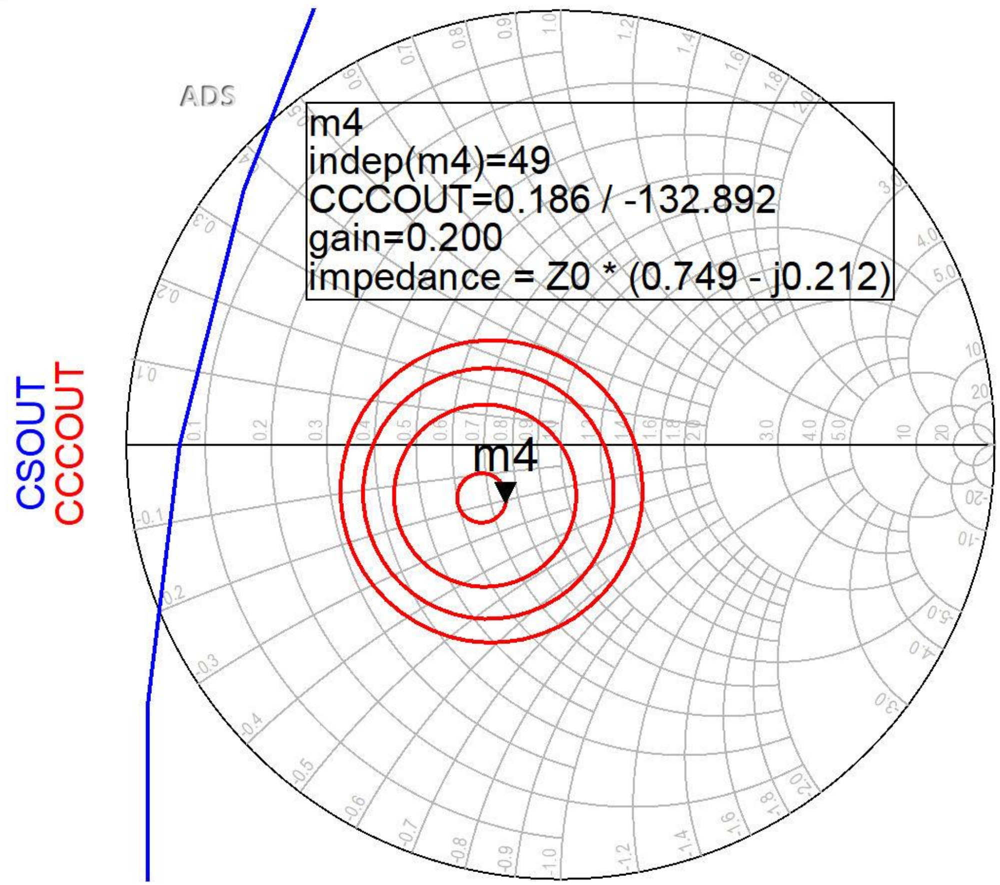
$$\boxed{\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}}$$

Input matching circuit



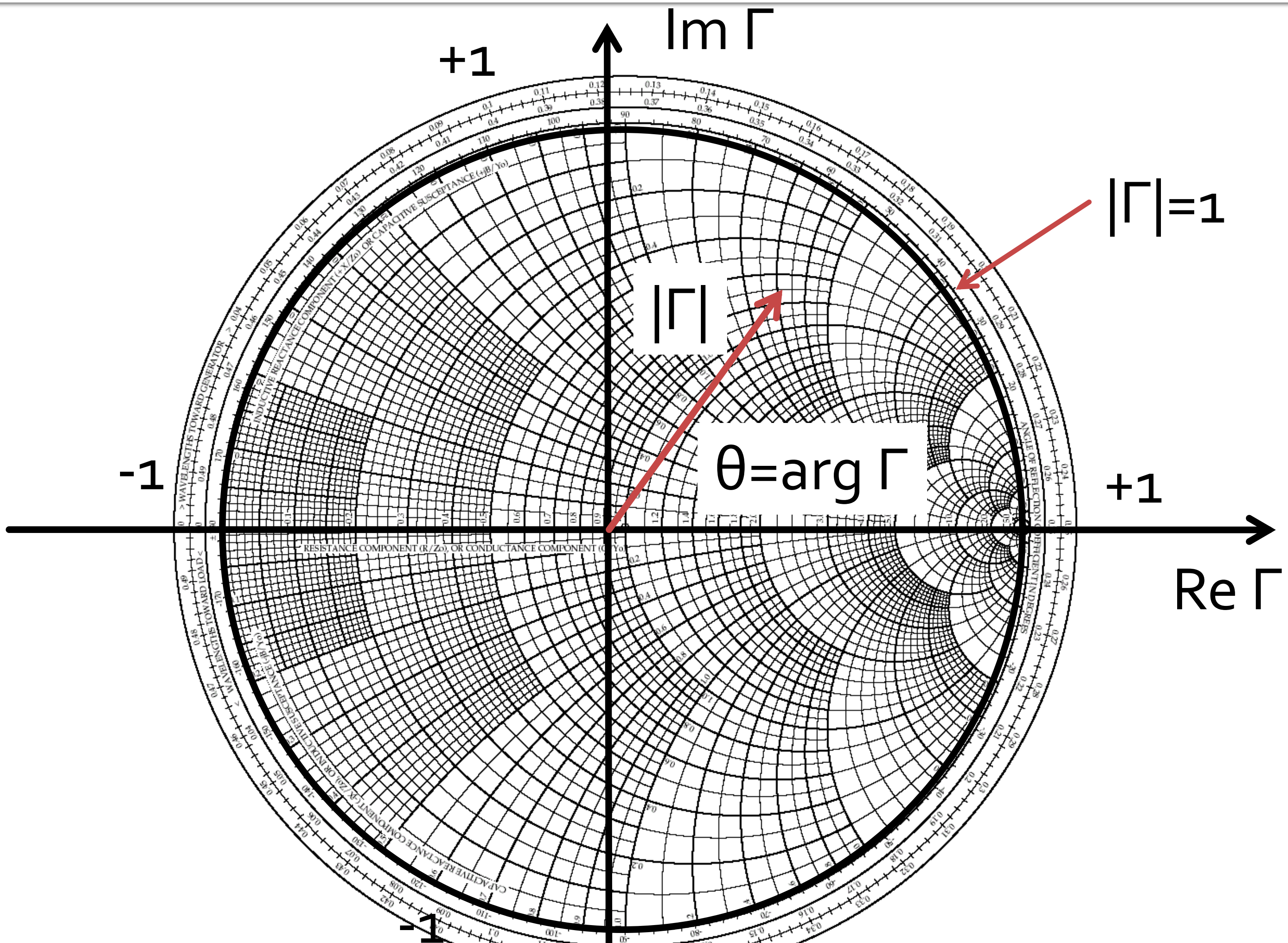
- If we can afford a 1.2dB decrease of the input gain for better NF,Q (Gs = 1 dB), position m1 above is better
- We obtain better (smaller) NF

Output matching circuit

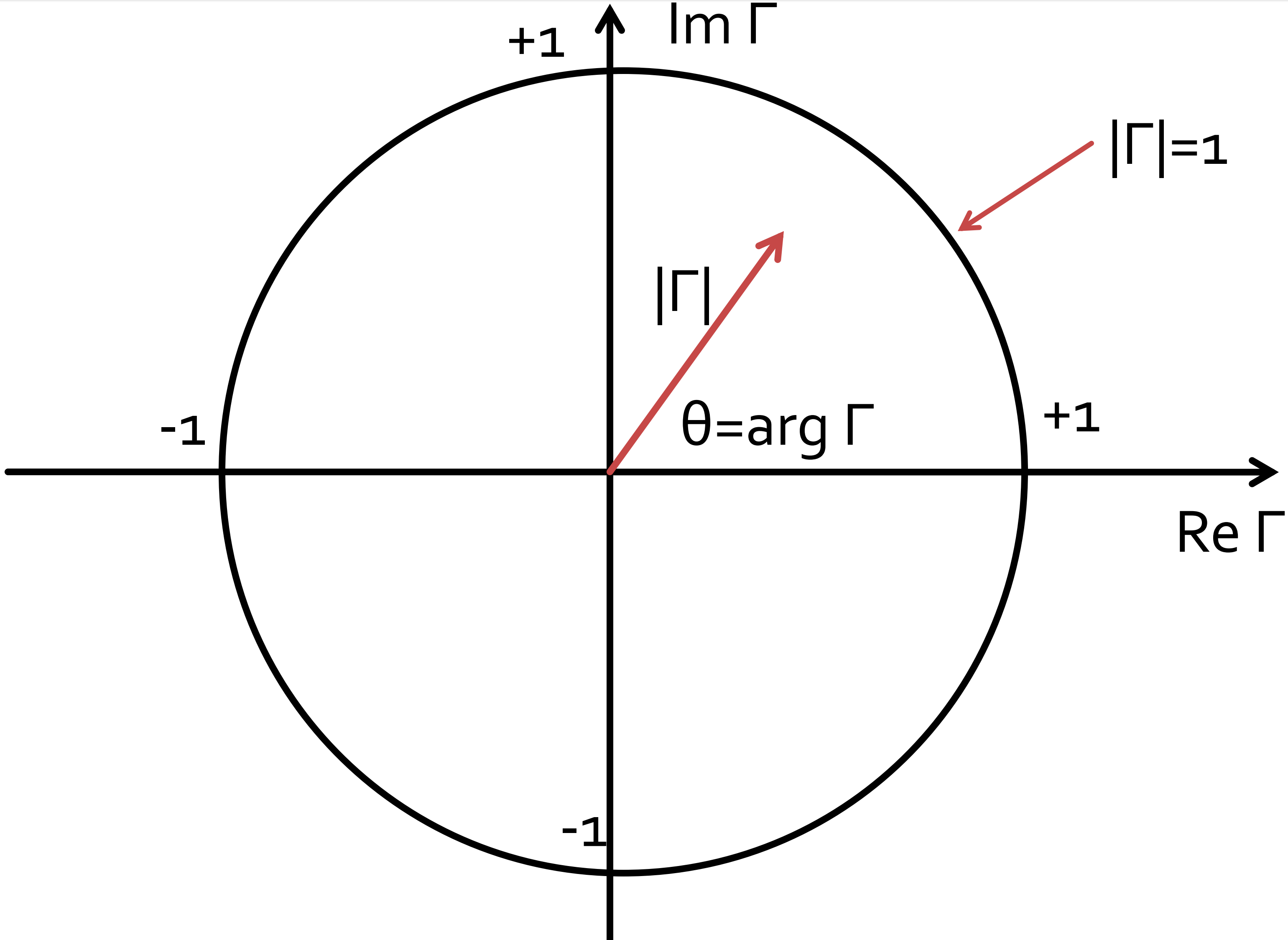


- output constant gain circles CCCOUT: -0.4dB, -0.2dB, 0dB, +0.2dB
- the lack of noise restrictions allows optimization for better gain (close to maximum – position m4)

The Smith Chart



The Smith Chart



Microwave Filters

Microwave Filters

- Two ways of implementing filters in microwave frequency range
  - microwave specific structures (coupled lines, dielectric resonators, periodic structures)
  - **filter synthesis** with lumped elements followed by implementation with transmission lines
- the first strategy leads to more efficient filters but:
  - has lower generality
  - design is often difficult (lack of analytical relationships)

Filter synthesis

- Filter is designed with lumped elements (L/C) followed by implementation with distributed elements (transmission lines)
  - general
  - analytical relationships easy to implement on the computer
  - efficient
- The preferred procedure is **insertion loss method**

Insertion loss method

$$P_{LR} = \frac{P_S}{P_L} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

- $|\Gamma(\omega)|^2$ is an even function of ω

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

- Choosing M and N polynomials appropriately leads to a filter with a completely specified frequency response

Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
  - in the passband (pass all frequencies)
  - in the stopband (reject all frequencies)

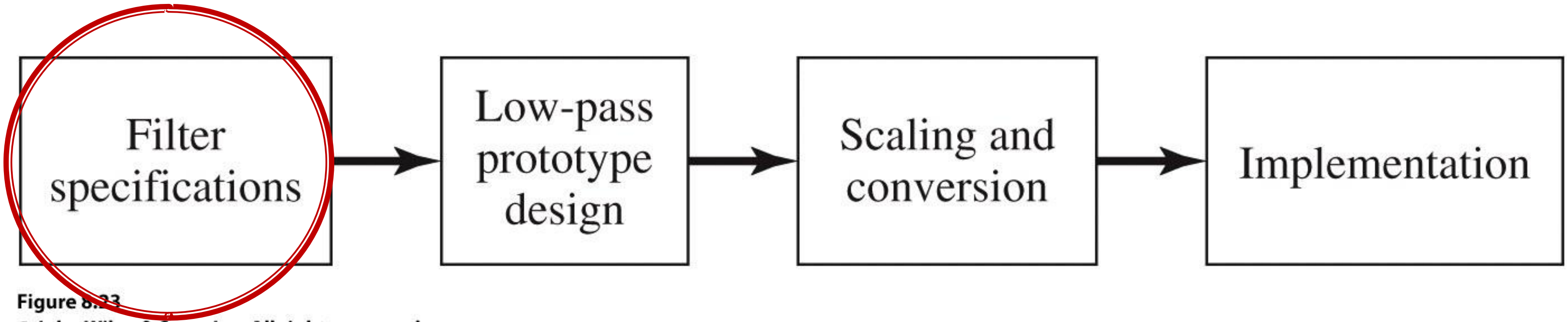
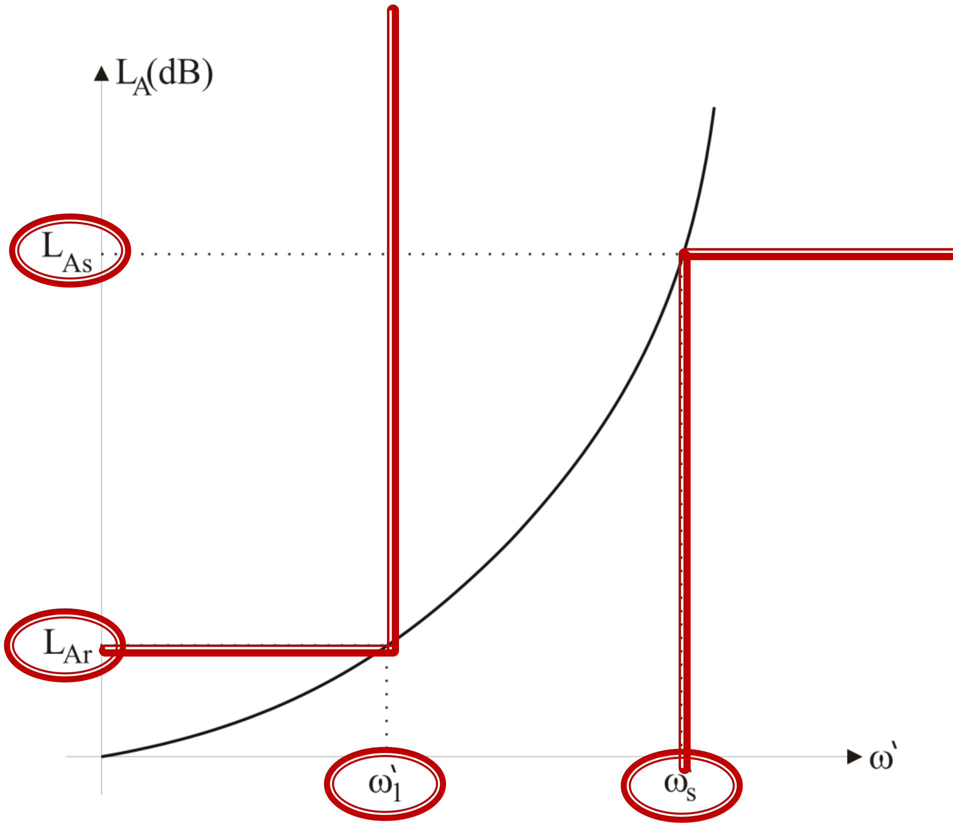


Figure 8.23

Filter specifications

- Attenuation
  - in passband
  - in stopband
  - most often in **dB**
- Frequency range
  - passband
  - stopband
  - cutoff frequency ω_1' usually normalized (**= 1**)



Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
  - low-pass, high-pass, bandpass, or bandstop

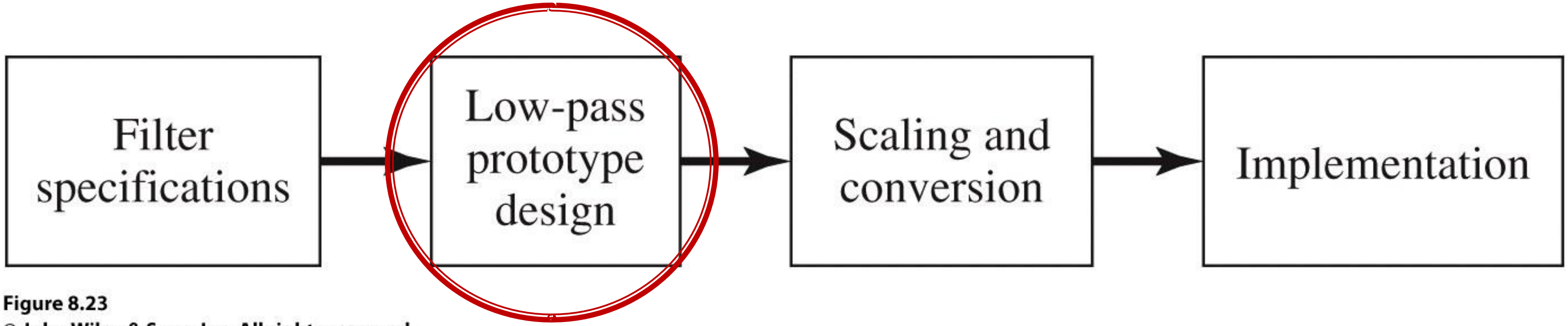


Figure 8.23

Practical low-pass prototypes responses

- **Maximally flat filters** (Butterworth, binomial): provide the flattest possible passband response
- **Equal ripple filters** (Chebyshev): provide a sharper cutoff but the passband response will have ripples
- **Elliptic function filters**, they have equal-ripple responses in the passband as well as in the stopband,
- **Linear phase filters**, offer linear phase response in the passband to avoid signal distortion (important in some applications)

Maximally Flat/Equal ripple LPF Prototype

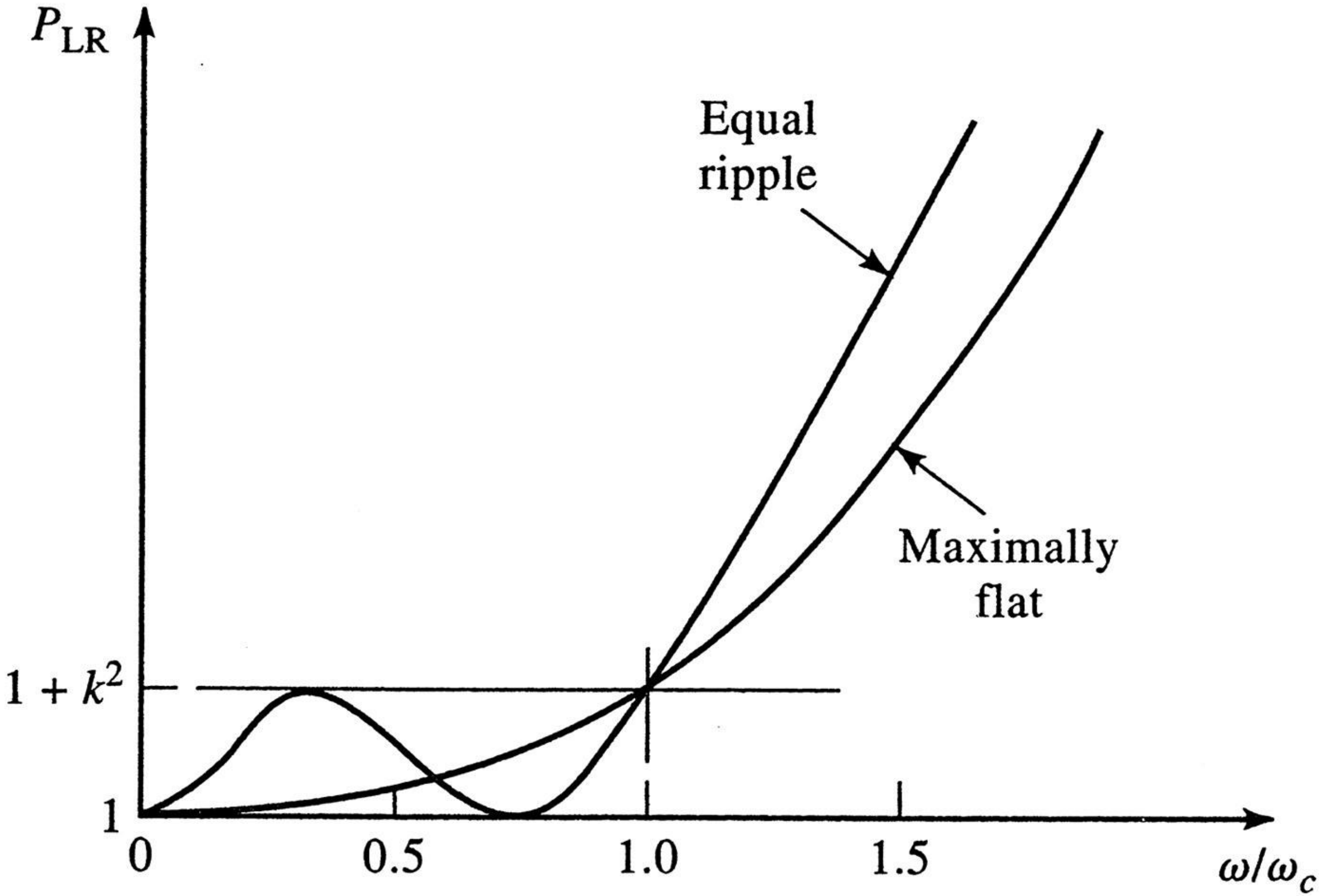


Figure 8.21

Elliptic function LPF Prototype

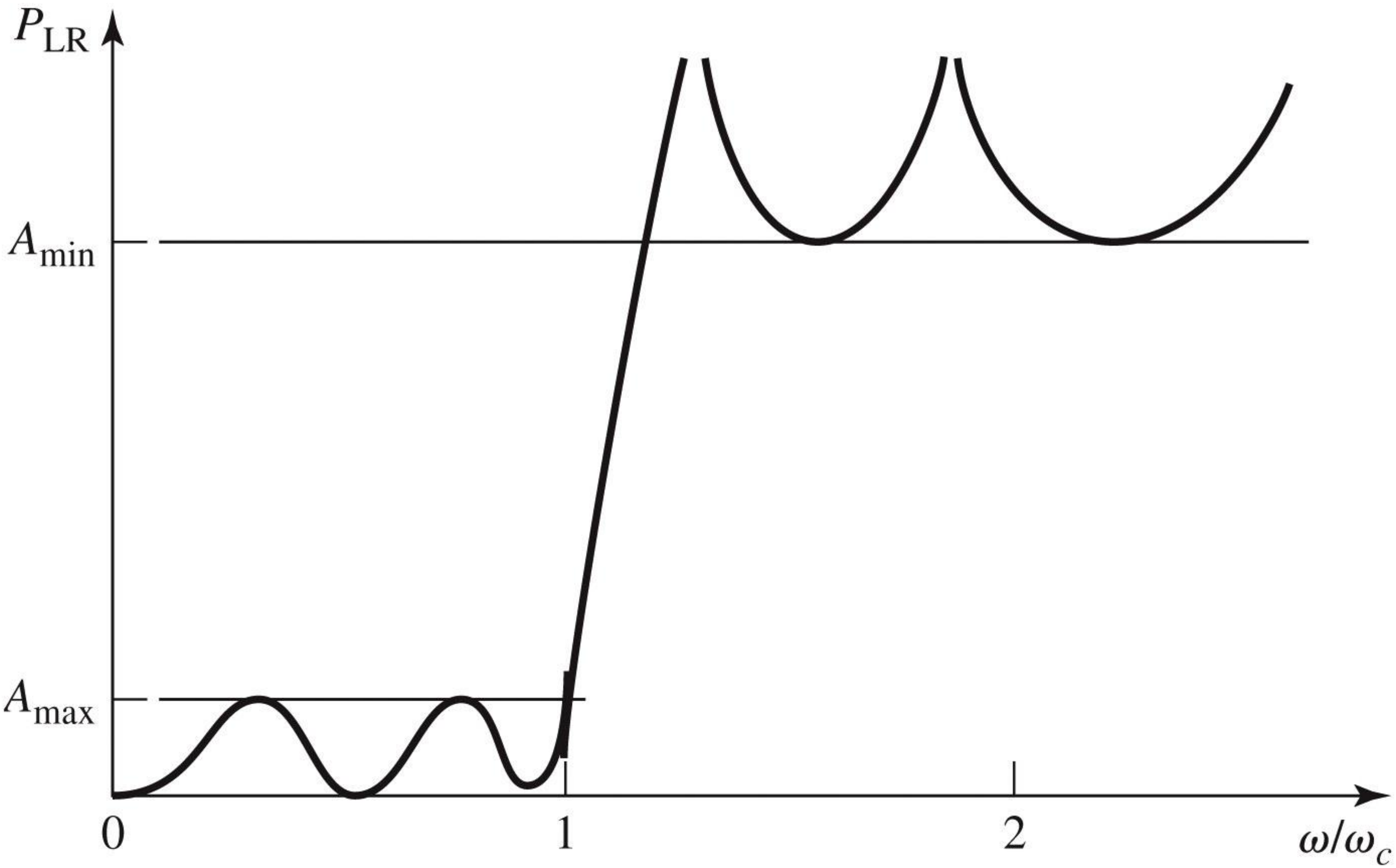


Figure 8.22

Maximally Flat LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot \left(\frac{\omega}{\omega_c} \right)^{2N}$$

- For $\omega >> \omega_c$

$$P_{LR} \approx k^2 \cdot (\omega / \omega_c)^{2N}$$

- attenuation increases at a rate of 20·N dB/decade
- k gives the attenuation at cutoff frequency (3dB cutoff imposes k = 1)

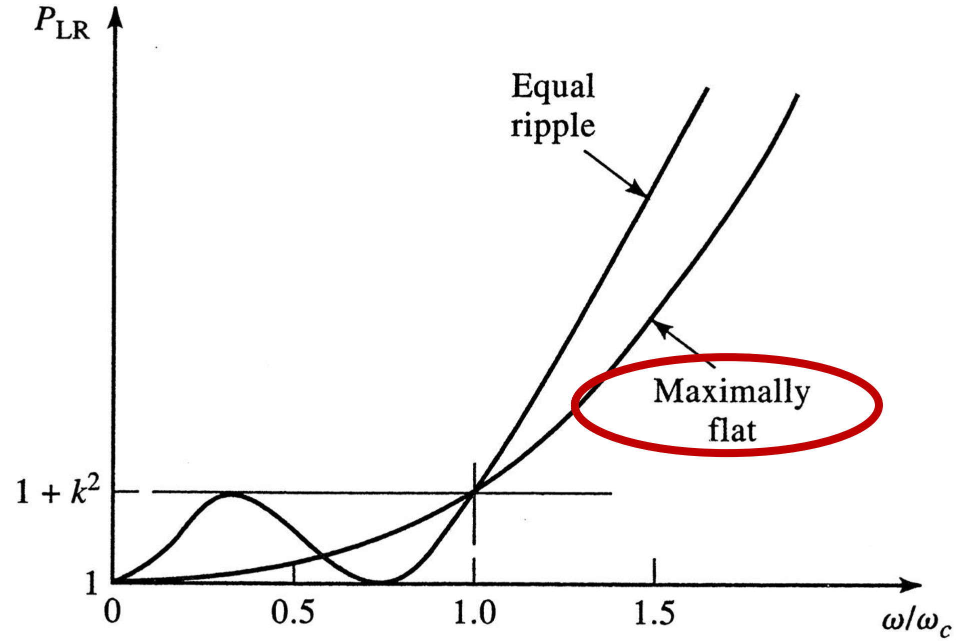


Figure 8.21

Equal Ripple LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot T_N^2\left(\frac{\omega}{\omega_c}\right)$$

- For $\omega >> \omega_c$

$$P_{LR} \approx \frac{k^2}{4} \cdot \left(\frac{2 \cdot \omega}{\omega_c}\right)^{2N}$$

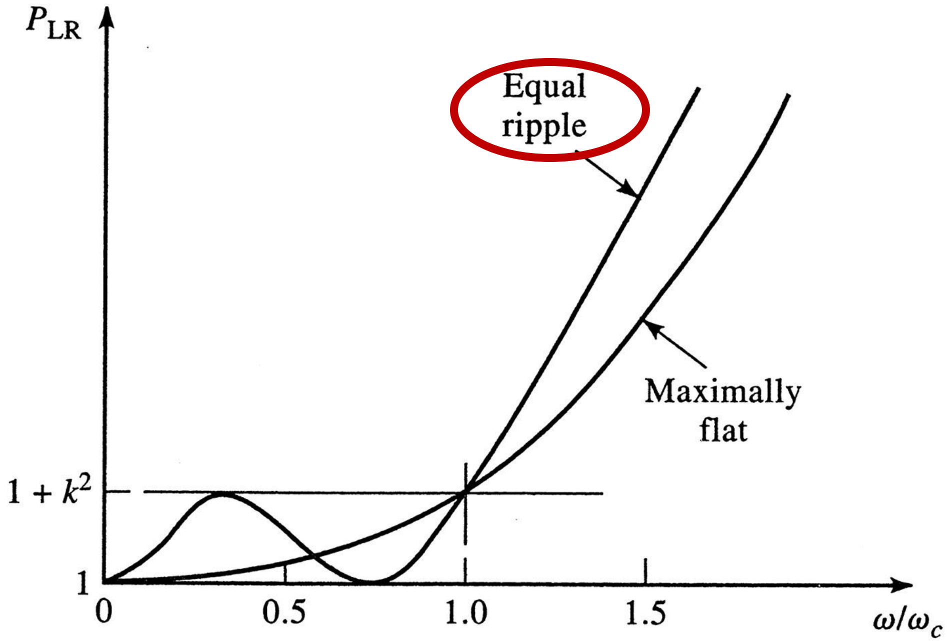


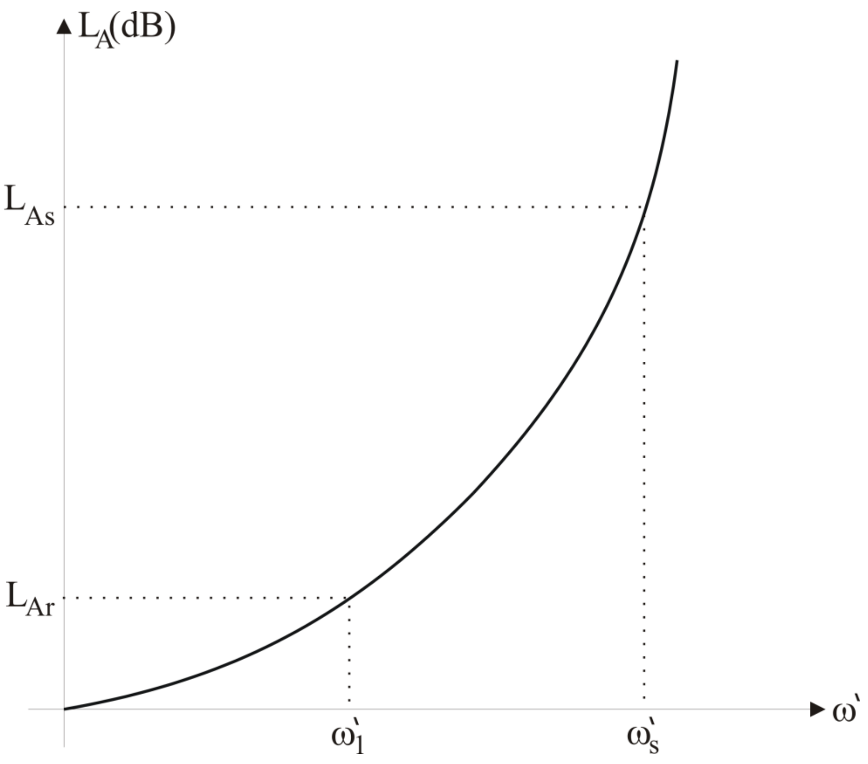
Figure 8.21

- attenuation increases at a rate of 20·N dB/decade (**also**)
- attenuation is $(2^{2N})/4$ greater than the binomial response at any given frequency where $\omega >> \omega_c$
- the passband ripples: $1 + k^2$, k gives the ripple

Order (N) of the Maximally Flat filter

$$n \geq \frac{\log\left(\dfrac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}\right)}{2\cdot\log\dfrac{\omega'_s}{\omega'_1}}$$

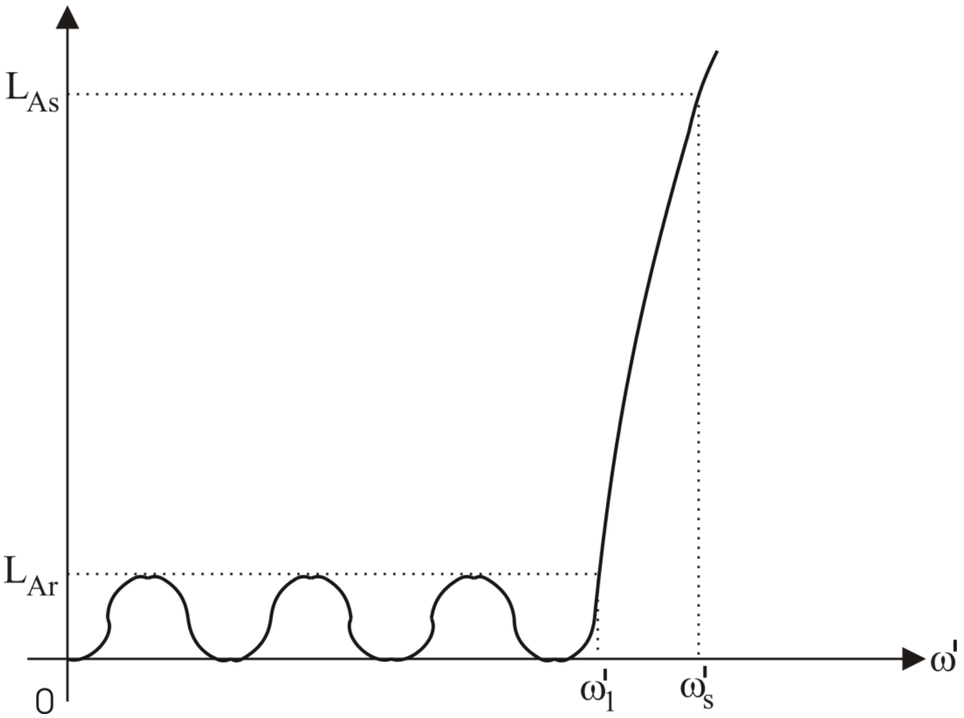
- **!**attenuations in **dB**



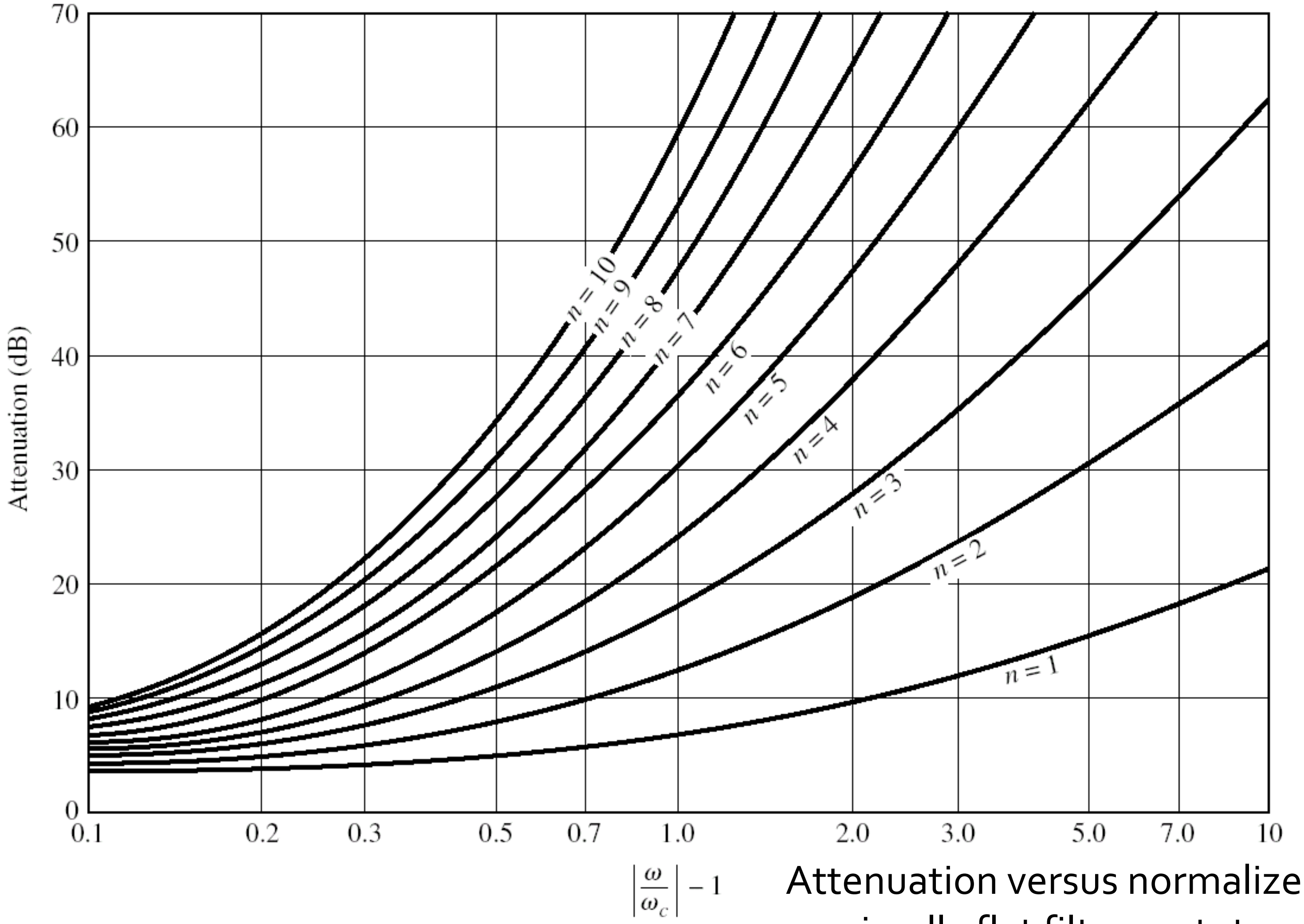
Order (N) of the Equal Ripple filter

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}}\right)}{\cosh^{-1}\left(\dfrac{\omega'_s}{\omega'_1}\right)}$$

- !attenuations in **dB**

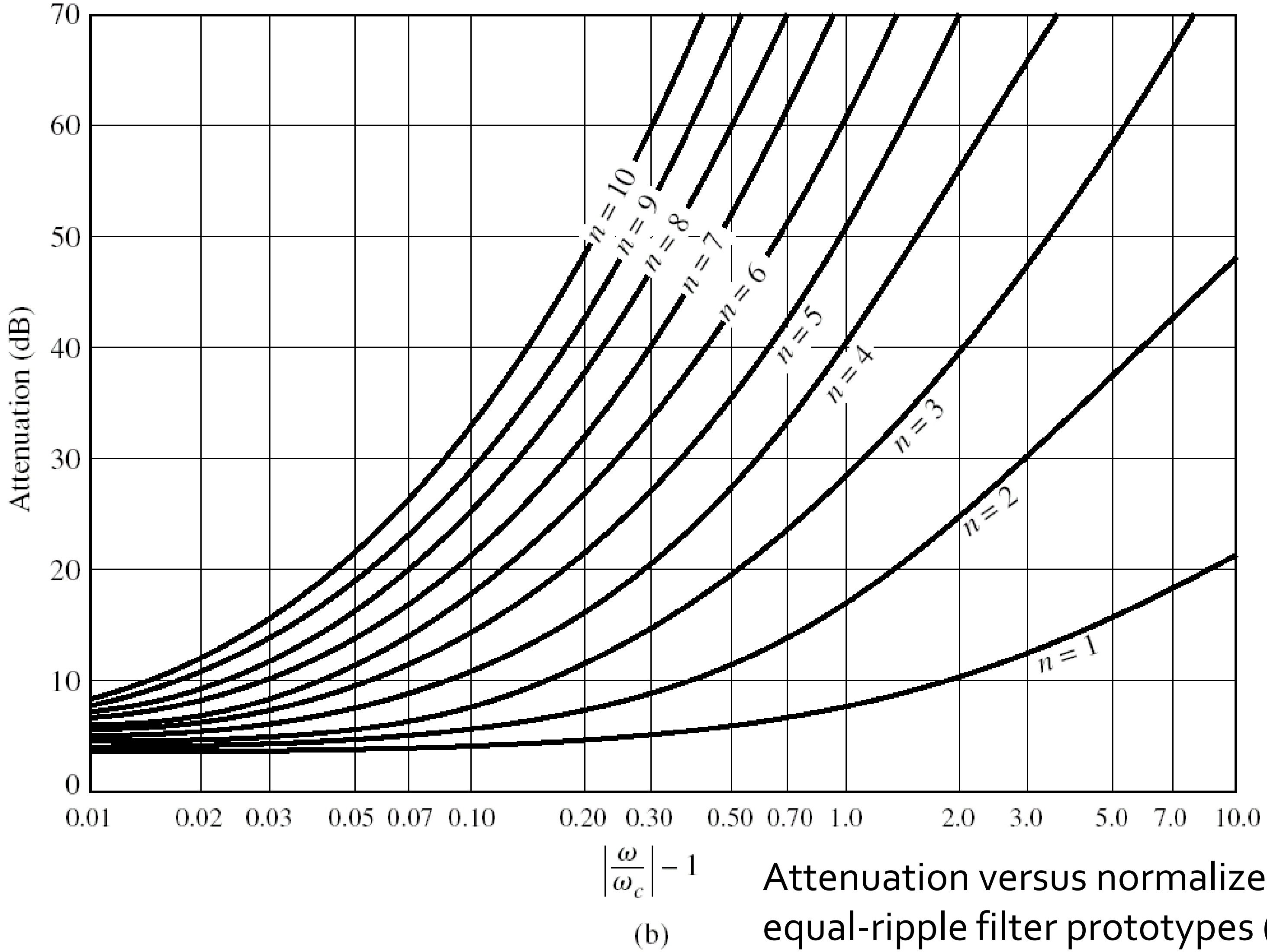


Maximally flat filter prototypes



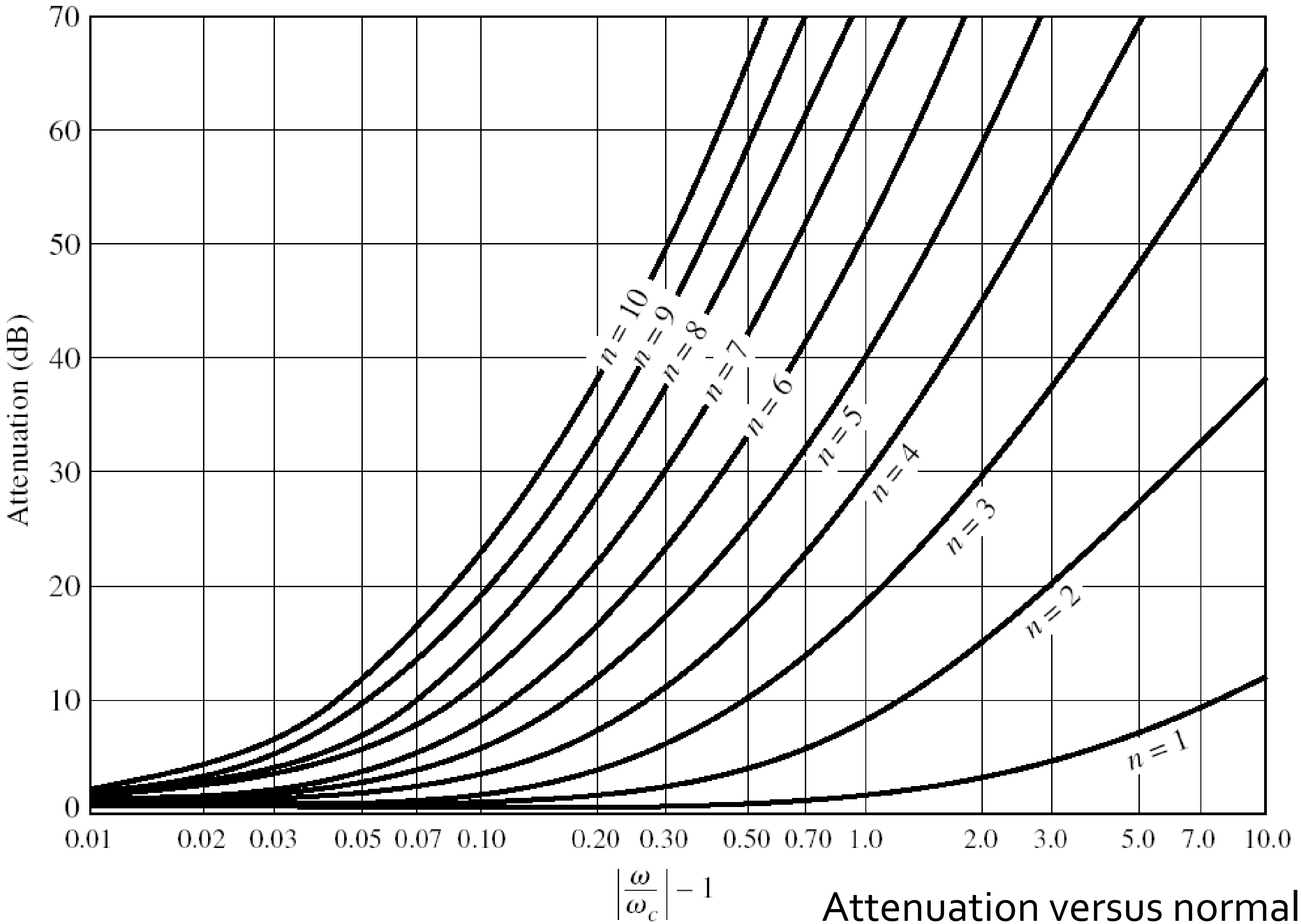
Attenuation versus normalized frequency for maximally flat filter prototypes

3 dB Equal-ripple filter prototypes



Attenuation versus normalized frequency for equal-ripple filter prototypes (3dB)

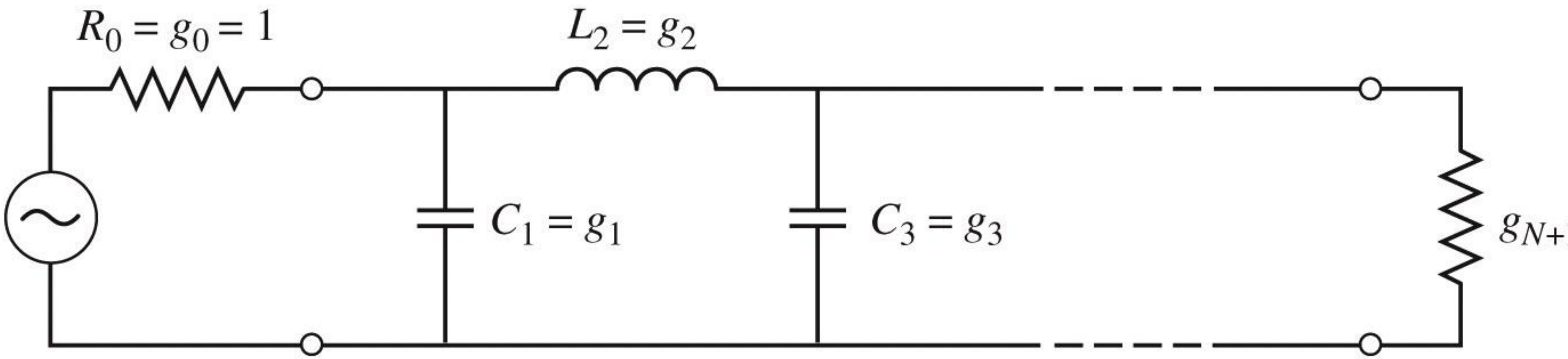
0.5 dB Equal-ripple filter prototypes



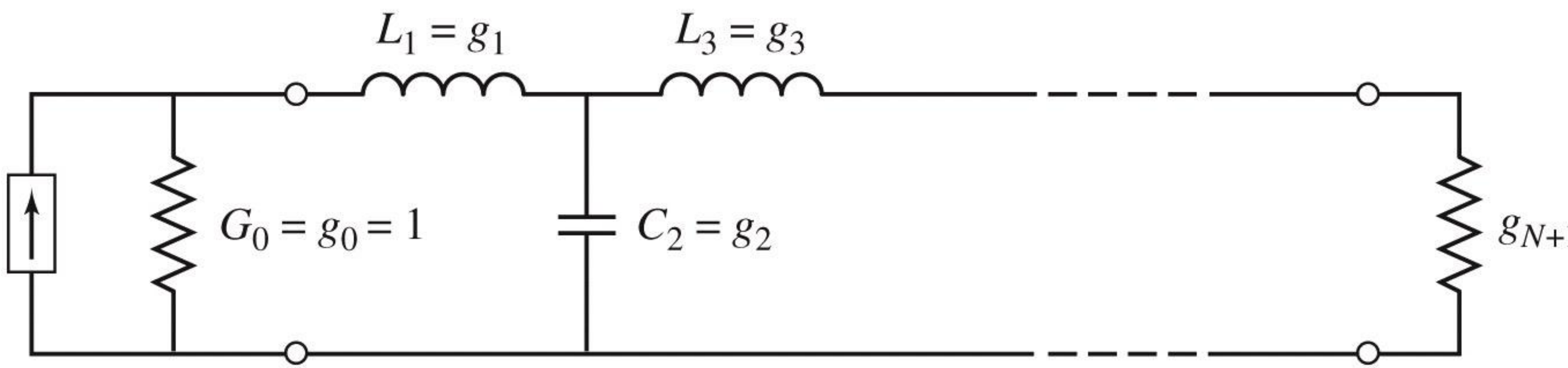
(a)

Attenuation versus normalized frequency for equal-ripple filter prototypes (0.5dB)

Prototype Filters



(a)



(b)

Prototype Filters

- Prototype filters are:
  - Low-Pass Filters (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω
- The number of reactive elements (L/C) is the order of the filter (N)
- Reactive elements are alternated: series L / shunt C
- There two prototypes with the same response, a prototype beginning with a shunt C element, and a prototype beginning with a series L element

Prototype Filters

- We define filter parameters g_i, $i=0,N+1$
- g_i are the element values in the prototype filter

$$g_0 = \begin{cases} \text{generator resistance } R'_0 \ \ if \ \ g_1 = C'_1 \\ \text{generator conductance } G'_0 \ \ if \ \ g_1 = L'_1 \end{cases}$$

$$g_k\Big|_{k=\overline{1,N}} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance } R'_{N+1} \ \ if \ \ g_N = C'_N \\ \text{load conductance } G'_{N+1} \ \ if \ \ g_N = L'_N \end{cases}$$

Maximally Flat LPF Prototype

- Formulas for filter parameters

$$g_0 = 1$$

$$g_k = 2 \cdot \sin\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N}\right] \quad , \quad k = 1, N$$

$$g_{N+1} = 1$$

Maximally Flat LPF Prototype

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Table 8.3

Equal-ripple LPF Prototype

- Formulas for filter parameters (iterative)

$$a_k = \sin\left[\frac{(2\cdot k-1)\cdot\pi}{2\cdot N}\right] \quad , \quad k=1,N$$

$$\beta = \ln\left(\coth\frac{L_{Ar}}{17.37}\right)$$

$$\gamma = \sinh\left(\frac{\beta}{2\cdot N}\right)$$

$$b_k = \gamma^2 + \sin^2\left(\frac{k\cdot\pi}{N}\right) \quad , \quad k=1,N$$

$$g_1 = \frac{2\cdot a_1}{\gamma}$$

$$g_k = \frac{4\cdot a_{k-1}\cdot a_k}{b_{k-1}\cdot g_{k-1}} \quad , \quad k=2,N$$

$$g_{N+1} = \begin{cases} 1 & for\, odd\ N \\ \coth^2\left(\frac{\beta}{4}\right) & for\, even\ N \end{cases}$$

TABLE 8.4 **Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)**

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass.,1980, with permission.

- For even N order of the filter (N = 2, 4, 6, 8 ...) equal-ripple filters **must** closed by a load impedance **$g_{N+1} \neq 1$**
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial ...) to **$g_L = 1$**

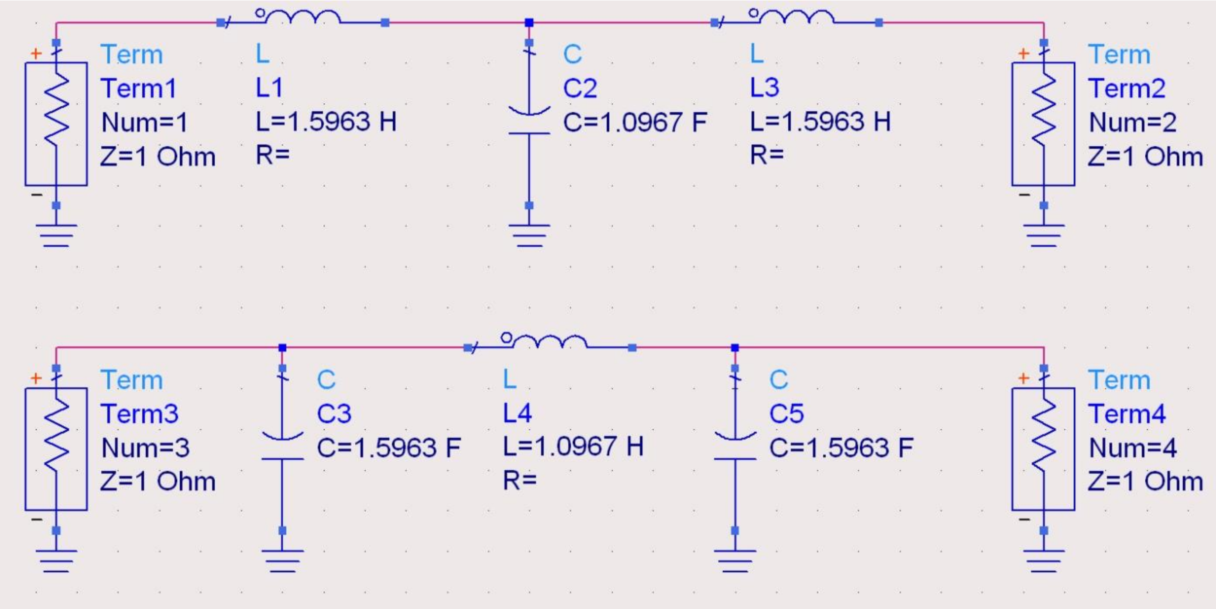
Table 8.4

Example

- Design a **3rd order** ~~bandpass~~ filter with **0.5 dB ripples** in passband. ~~The center frequency of the filter should be 1 GHz. The fractional bandwidth of the passband should be 10%, and the impedance 50Ω~~.

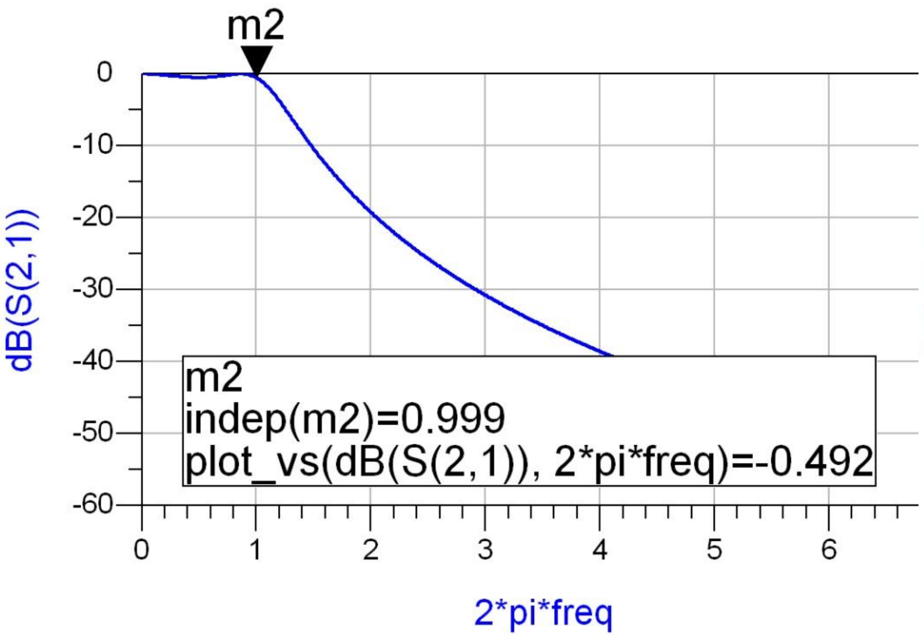
LPF Prototype

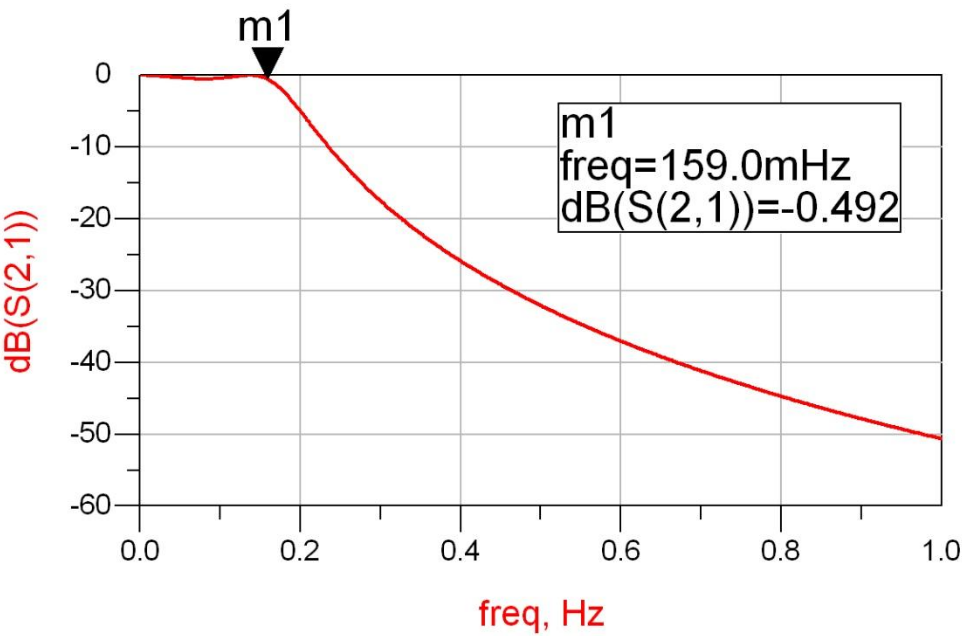
- 0.5dB equal-ripple table or design formulas:
  - $g_1 = 1.5963 = L_1/C_3$,
  - $g_2 = 1.0967 = C_2/L_4$,
  - $g_3 = 1.5963 = L_3/C_5$,
  - $g_4 = 1.000 = R_L$



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Impedance and Frequency Scaling

- After computing prototype filter's elements:
  - Low-Pass Filters (LPF)
  - cutoff frequency $\omega_o = 1$ rad/s ($f_o = 0.159$ Hz)
  - connected to a source with $R = 1\Omega$
- component values can be scaled in terms of impedance and frequency

Impedance and Frequency Scaling

- LPF Prototype is only used as an intermediate step
  - Low-Pass Filter (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω

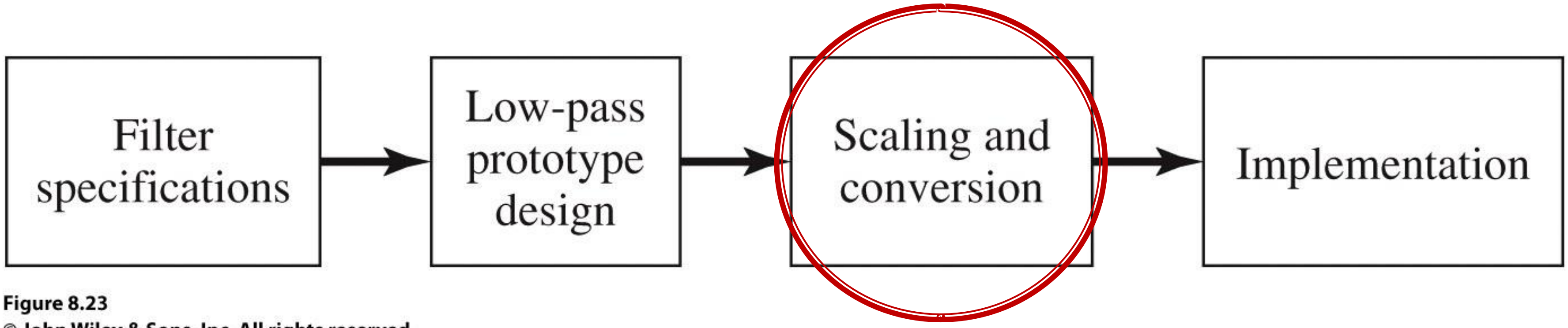


Figure 8.23

Impedance Scaling

- To design a filter which will work with a source resistance of R_o we multiplying all the impedances of the prototype design by R_o ("**'**" denotes scaled values)

$$R_s' = R_0 \cdot (R_s = 1)$$

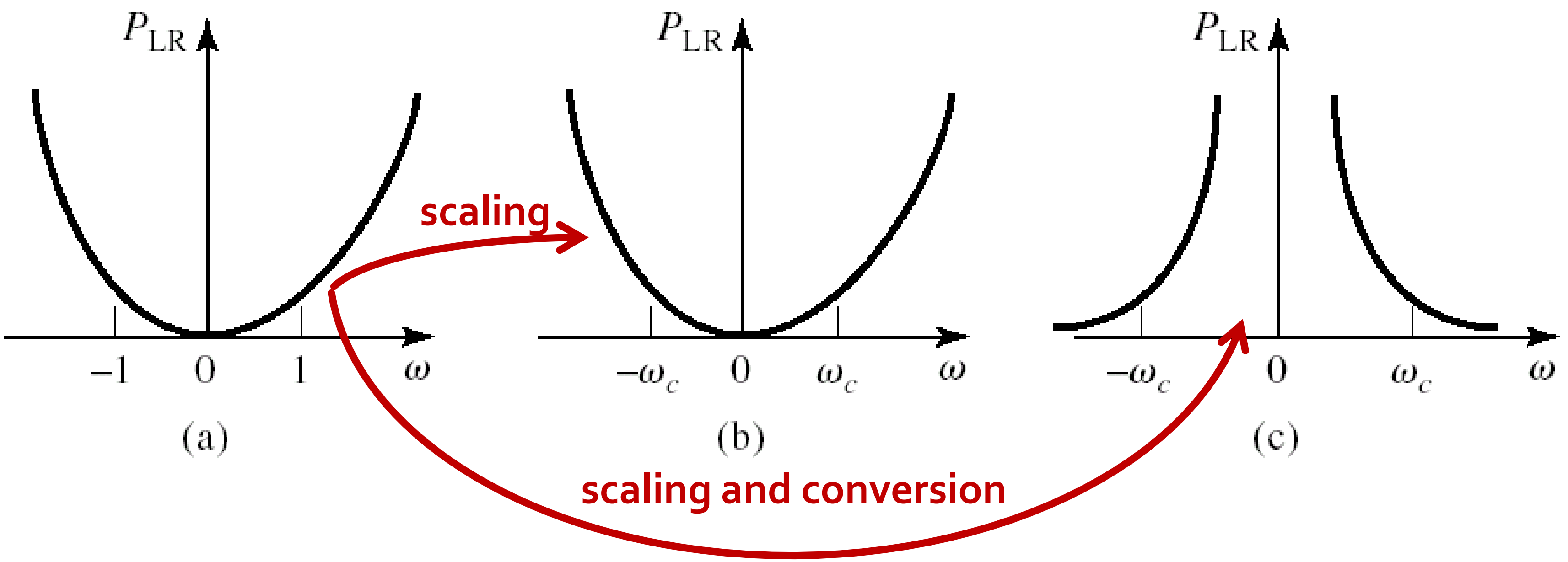
$$R_L' = R_0 \cdot R_L$$

$$L' = R_0 \cdot L$$

$$C' = \frac{C}{R_0}$$

Frequency Scaling

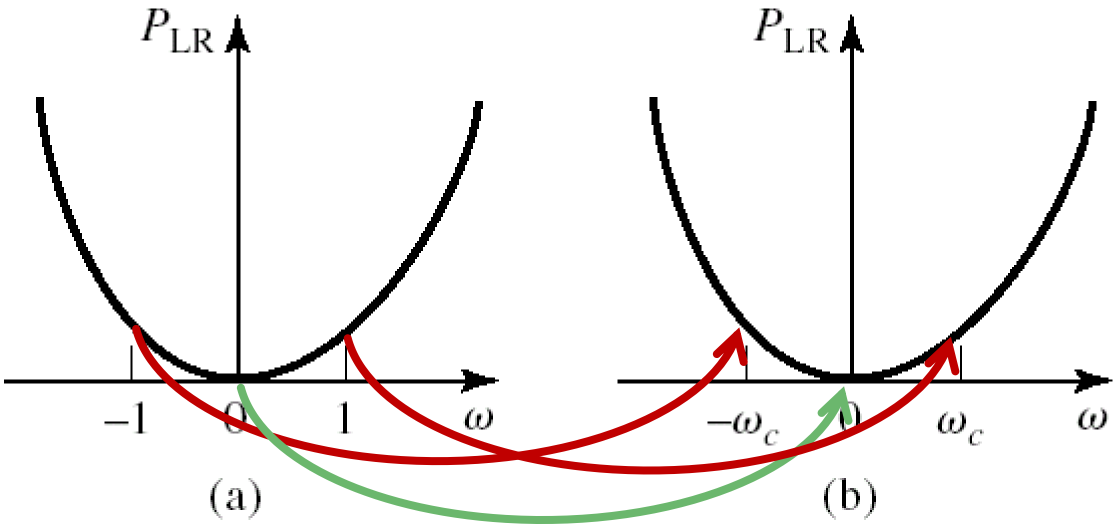
- changing the cutoff frequency – (fig. b)
- changing the type (for example LPF → HPF – fig. c) requires also conversion



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype from unity to ω_c we insert a variable change

$$\omega \leftarrow \frac{\omega}{\omega_c}$$



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype we insert a variable substitution:

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

- Equivalent to the widening of the power loss filter response

$$P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right)$$

$$j \cdot X_k = j \cdot \frac{\omega}{\omega_c} \cdot L_k = j \cdot \omega \cdot L'_k \qquad j \cdot B_k = j \cdot \frac{\omega}{\omega_c} \cdot C_k = j \cdot \omega \cdot C'_k$$

Frequency Scaling LPF → LPF

- New element values for frequency scaling:

$$L_k' = \frac{L_k}{\omega_c} \qquad C_k' = \frac{C_k}{\omega_c}$$

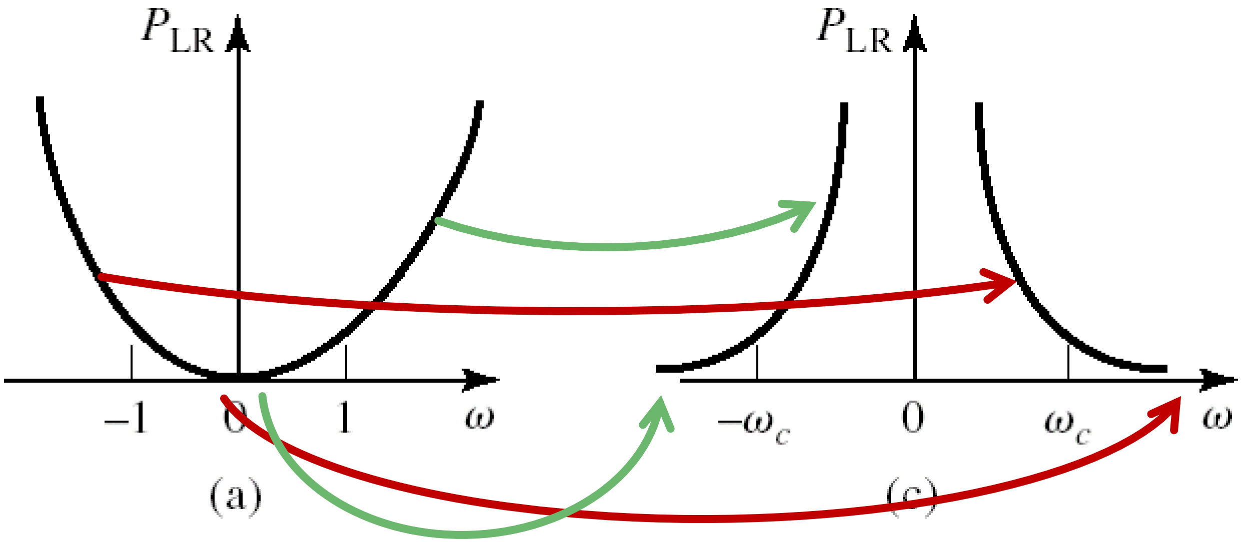
- When both impedance and frequency scaling are required:

$$L_k' = \frac{R_0 \cdot L_k}{\omega_c} \qquad C_k' = \frac{C_k}{R_0 \cdot \omega_c}$$

Low-pass to high-pass transformation LPF → HPF

- Variable substitution for LPF → HPF:

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$



High-pass transformation LPF → HPF

- Variable substitution for LPF → HPF :

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

$$j \cdot X_k = -j \cdot \frac{\omega_c}{\omega} \cdot L_k = \frac{1}{j \cdot \omega \cdot C'_k} \qquad j \cdot B_k = -j \cdot \frac{\omega_c}{\omega} \cdot C_k = \frac{1}{j \cdot \omega \cdot L'_k}$$

- Impedance scaling can be included

$$C'_k = \frac{1}{R_0 \cdot \omega_c \cdot L_k} \qquad L'_k = \frac{R_0}{\omega_c \cdot C_k}$$

- In the schematic series inductors must be replaced with series capacitors, and shunt capacitors must be replaced with shunt inductors

Bandpass Transformation LPF → BPF

- Variable substitution for LPF → BPF:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

- where we use the fractional bandwidth of the passband and the center frequency

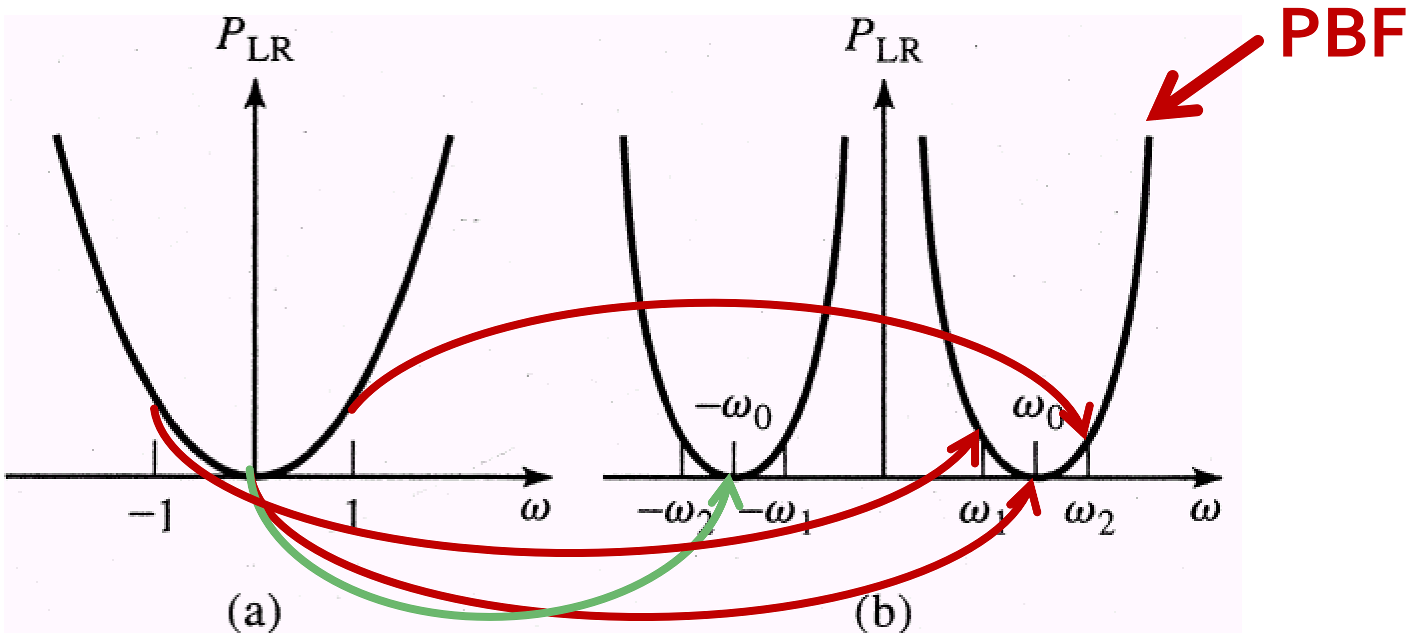
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \qquad \omega_0 = \sqrt{\omega_1 \cdot \omega_2}$$

Bandpass Transformation LPF → BPF

$$\omega=\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_0}{\omega_0}-\frac{\omega_0}{\omega_0}\right)=0 \qquad \omega=-\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{-\omega_0}{\omega_0}-\frac{\omega_0}{-\omega_0}\right)=0$$

$$\omega=\omega_1 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_1^2-\omega_0^2}{\omega_0\cdot\omega_1}\right)=-1$$

$$\omega=\omega_2 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_2^2-\omega_0^2}{\omega_0\cdot\omega_2}\right)=1$$



Bandpass Transformation LPF → BPF

$$j \cdot X_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot L_k = j \cdot \frac{\omega \cdot L_k}{\Delta \cdot \omega_0} - j \cdot \frac{\omega_0 \cdot L_k}{\Delta \cdot \omega} = j \cdot \omega \cdot L_k' - j \frac{1}{\omega \cdot C_k'}$$

$$j \cdot B_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot C_k = j \cdot \frac{\omega \cdot C_k}{\Delta \cdot \omega_0} - j \cdot \frac{\omega_0 \cdot C_k}{\Delta \cdot \omega} = j \cdot \omega \cdot C_k' - j \frac{1}{\omega \cdot L_k'}$$

- A series **inductor** in the prototype filter is transformed to a **series LC** circuit

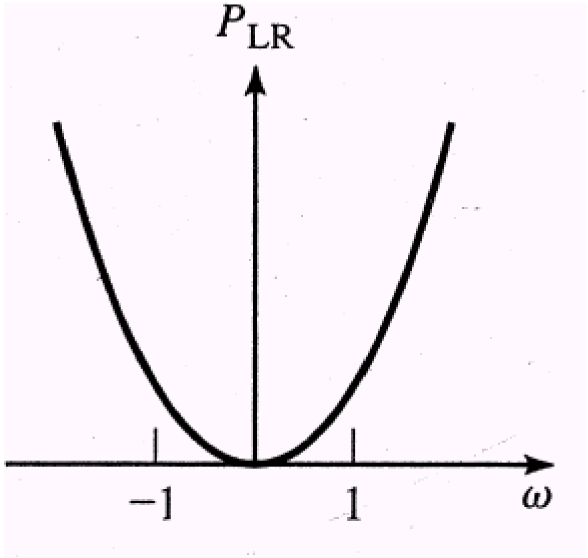
$$L_k' = \frac{L_k}{\Delta \cdot \omega_0} \qquad C_k' = \frac{\Delta}{\omega_0 \cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **shunt LC** circuit

$$L_k' = \frac{\Delta}{C_k \cdot \omega_0} \qquad C_k' = \frac{C_k}{\omega_0 \cdot \Delta}$$

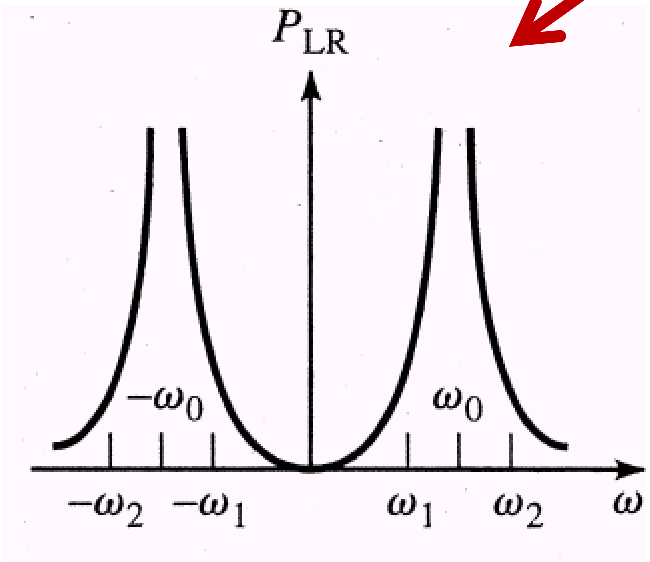
Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1} \qquad \omega = \omega_0 \rightarrow \frac{-\Delta}{\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)} = \frac{-\Delta}{\left(\frac{\omega_0}{\omega_0} - \frac{\omega_0}{\omega_0} \right)} \rightarrow \pm\infty$$



(a)

BSF



(c)

Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

- A series **inductor** in the prototype filter is transformed to a **shunt LC** circuit

$$L'_k = \frac{\Delta \cdot L_k}{\omega_0} \qquad C'_k = \frac{1}{\omega_0 \cdot \Delta \cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **series LC** circuit

$$L'_k = \frac{1}{\Delta \cdot \omega_0 \cdot C_k} \qquad C'_k = \frac{\Delta \cdot C_k}{\omega_0}$$

Summary of Prototype Filter Transformations

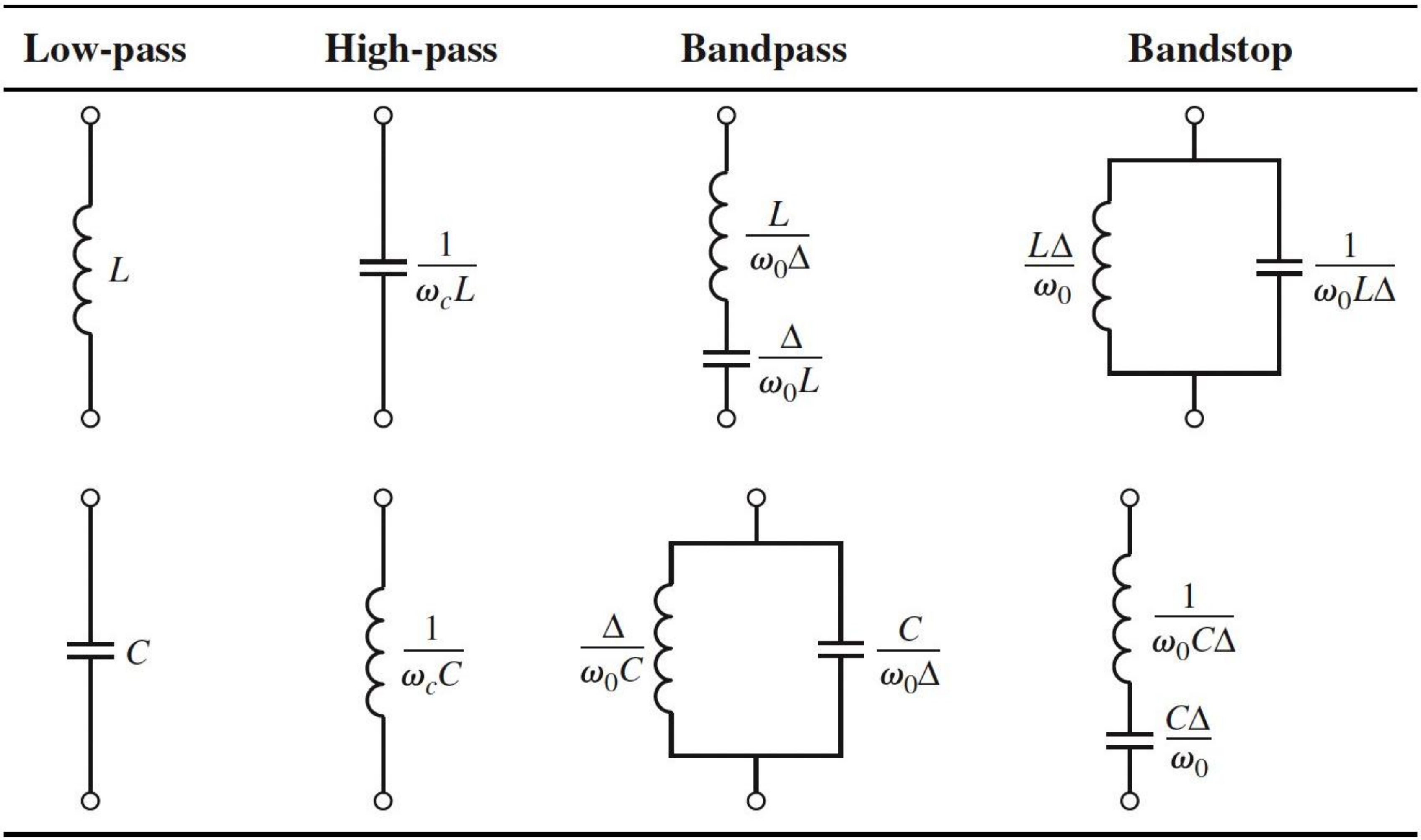


Table 8.6

Example

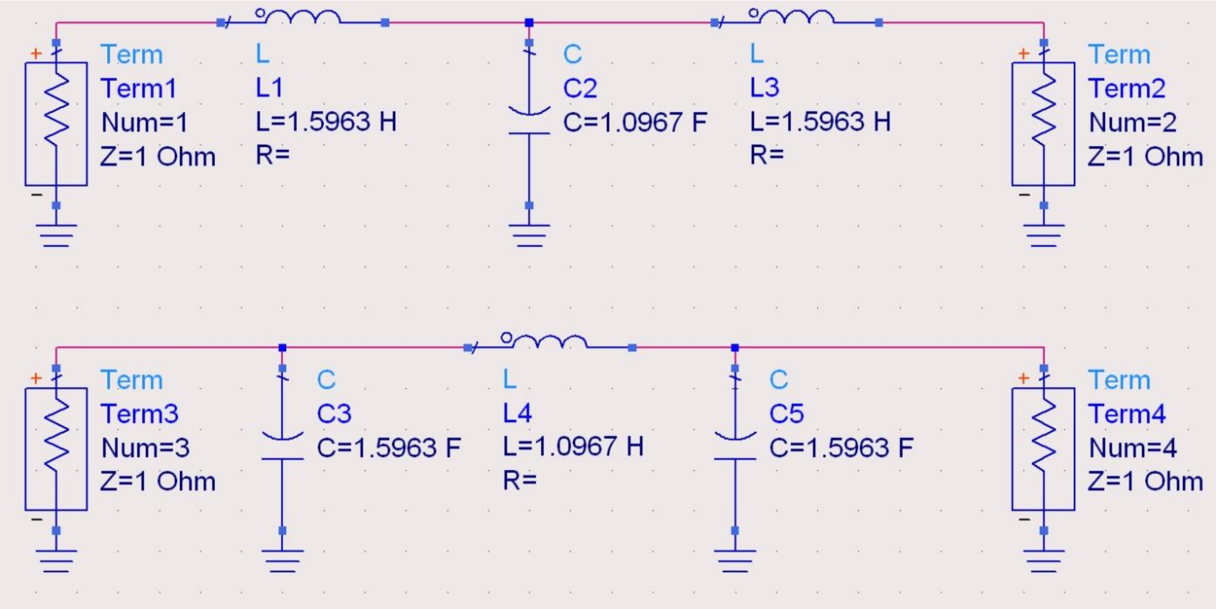
- Design a 3rd order **bandpass** filter with 0.5 dB ripples in passband. The **center frequency** of the filter should be 1 GHz. The **fractional bandwidth** of the passband should be 10%, and the **impedance** 50Ω.

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad / s$$

$$\Delta = 0.1$$

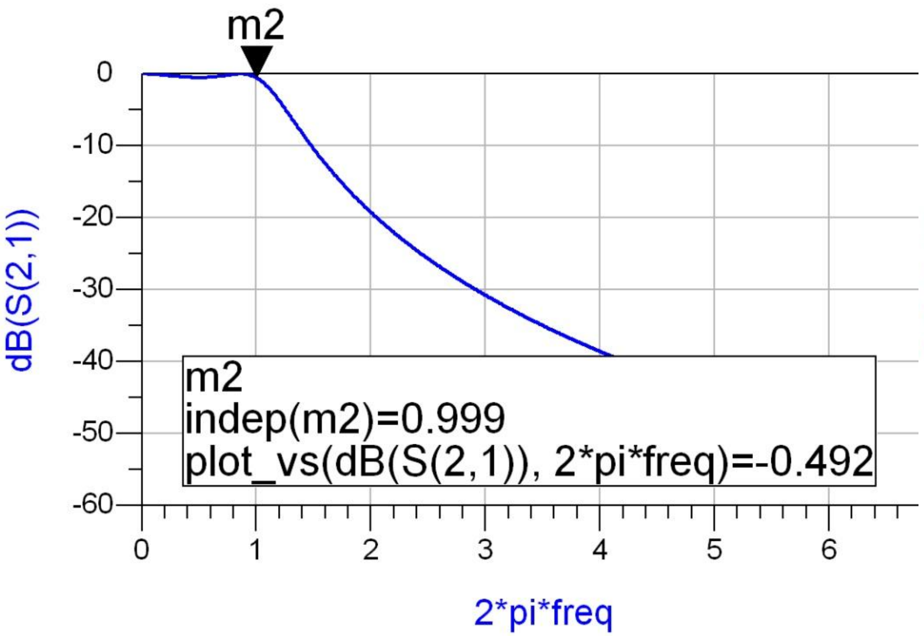
LPF Prototype

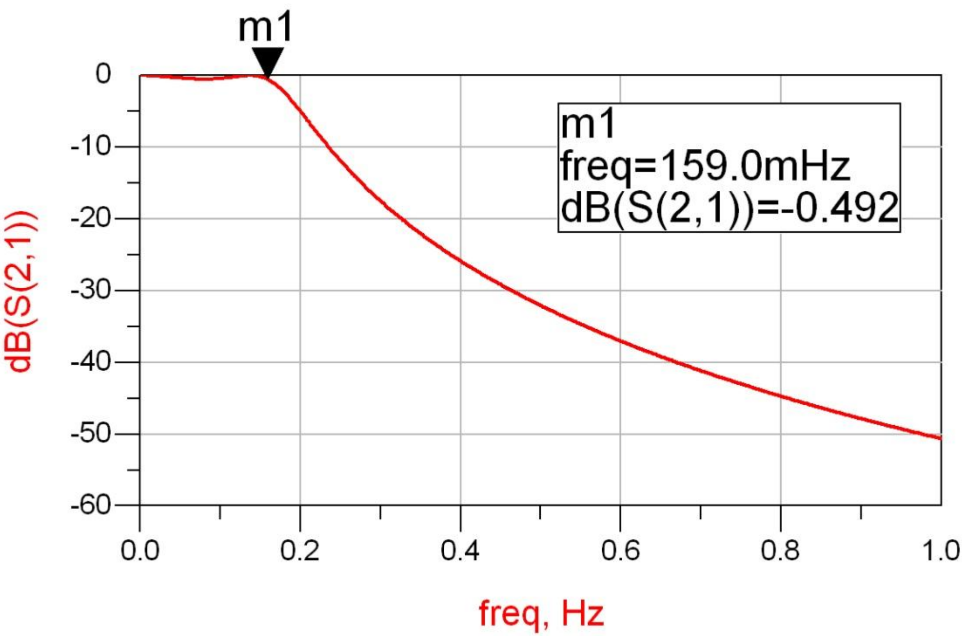
- 0.5dB equal-ripple table or design formulas:
  - $g1 = 1.5963 = L1/C3$,
  - $g2 = 1.0967 = C2/L4$,
  - $g3 = 1.5963 = L3/C5$,
  - $g4 = 1.000 = R_L$



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Bandpass Transformation / BPF

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad/s \qquad \Delta = \frac{\Delta\omega}{\omega_0} = \frac{\Delta f}{f_0} = 0.1 \qquad R_0 = 50\ \Omega$$

g1 = 1.5963 = L1, g3 = 1.5963 = L3,

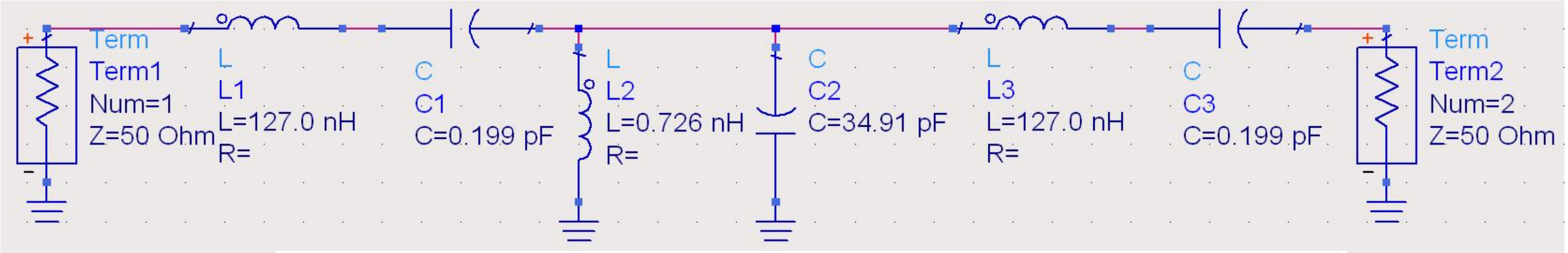
g2 = 1.0967 = C2, g4=1.000 =R_L

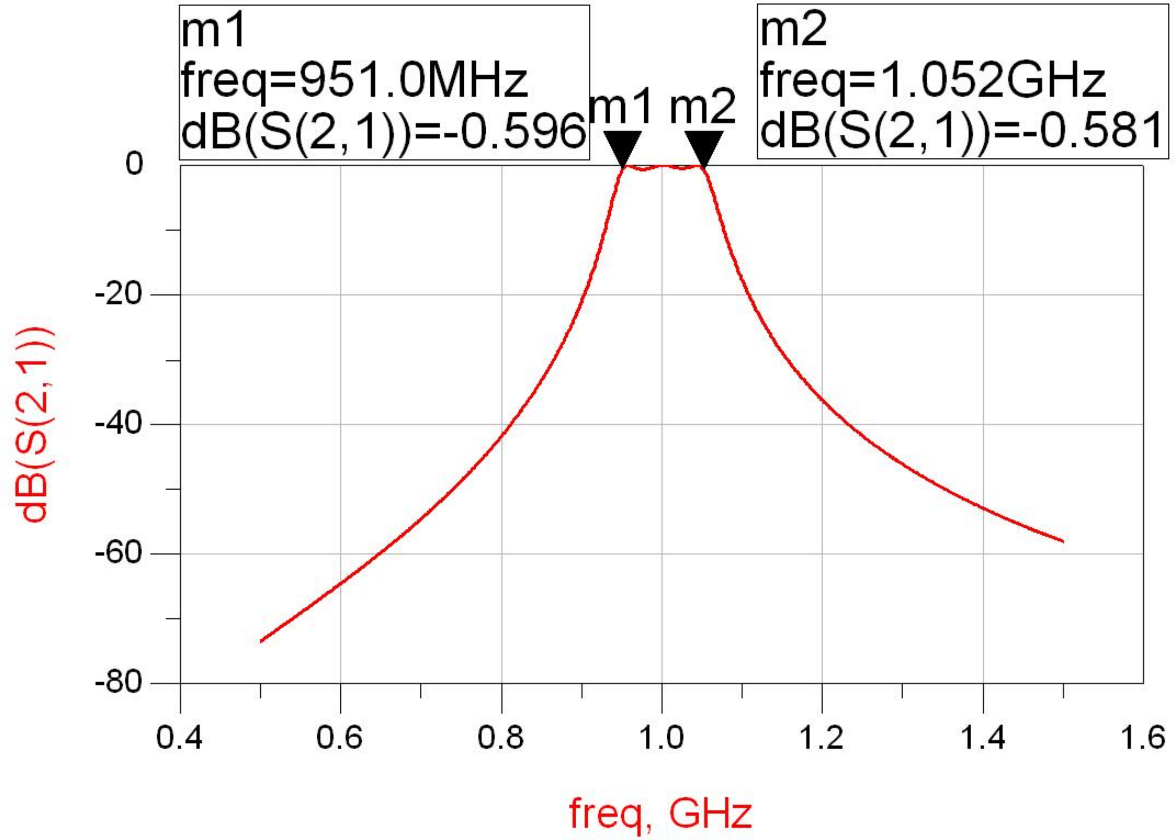
$$L_1' = \frac{L_1 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\, nH \qquad C_1' = \frac{\Delta}{\omega_0 \cdot L_1 \cdot R_0} = 0.199\, pF$$

$$L_2' = \frac{\Delta \cdot R_0}{\omega_0 \cdot C_2} = 0.726\, nH \qquad C_2' = \frac{C_2}{\Delta \cdot \omega_0 \cdot R_0} = 34.91\, pF$$

$$L_3' = \frac{L_3 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\, nH \qquad C_3' = \frac{\Delta}{\omega_0 \cdot L_3 \cdot R_0} = 0.199\, pF$$

ADS


Term
Term1
Num=1
Z=50 Ohm
L
L1
L=127.0 nH
R=
C
C1
C=0.199 pF
L
L2
L=0.726 nH
R=
C
C2
C=34.91 pF
L
L3
L=127.0 nH
R=
C
C3
C=0.199 pF
Term
Term2
Num=2
Z=50 Ohm


m1
freq=951.0MHz
dB(S(2,1))=-0.596
m2
freq=1.052GHz
dB(S(2,1))=-0.581
m1 m2
0
-20
-40
-60
-80
dB(S(2,1))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
freq, GHz

Microwave Filters Implementation

Microwave Filters Implementation

- The lumped-element (L, C) filter design generally works well **only** at low frequencies (RF):
  - lumped-element inductors and capacitors are generally available only for a limited range of values, and can be difficult to implement at microwave frequencies
  - difficulty to obtain the (very low) required tolerance for elements

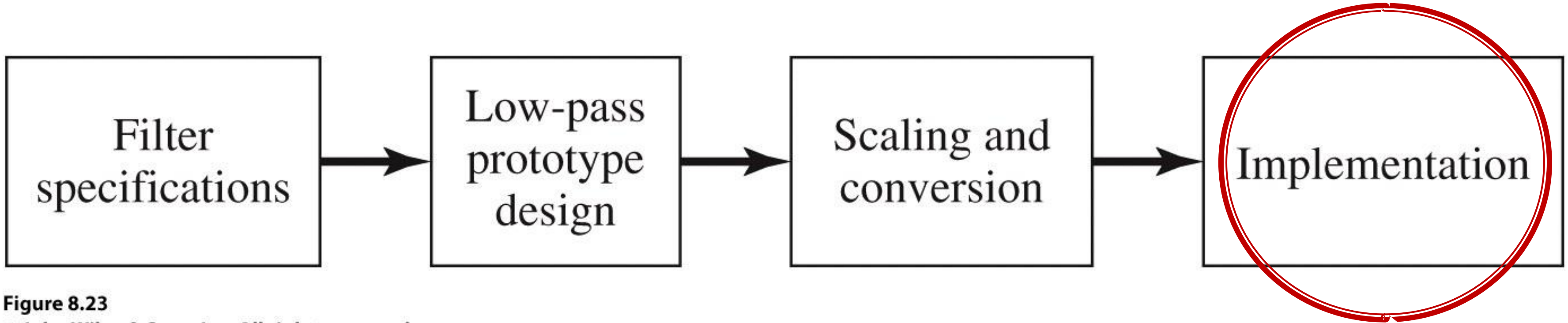


Figure 8.23

Richards' Transformation

- Impedance seen at the input of a line loaded with Z_L

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

- We prefer the load impedance to be:
  - open circuit ($Z_L = \infty$) $\quad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$
  - short circuit ($Z_L = 0$) $\quad Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$
- Input impedance is:
  - capacitive $\quad Z_{in,oc} = j \cdot X_C = \frac{1}{j \cdot B_C} \qquad Z_0 \leftrightarrow \frac{1}{C} \quad \tan \beta \cdot l \leftrightarrow \omega$
  - inductive $\quad Z_{in,sc} = j \cdot X_L \qquad Z_0 \leftrightarrow L \quad \tan \beta \cdot l \leftrightarrow \omega$

Richards' Transformation

- Variable substitution

$$\Omega = \tan \beta \cdot l = \tan\left(\frac{\omega \cdot l}{v_p}\right)$$

- With this variable substitution we define:
  - reactance of an inductor

$$j \cdot X_L = j \cdot \Omega \cdot L = j \cdot L \cdot \tan \beta \cdot l$$

  - susceptance of a capacitor

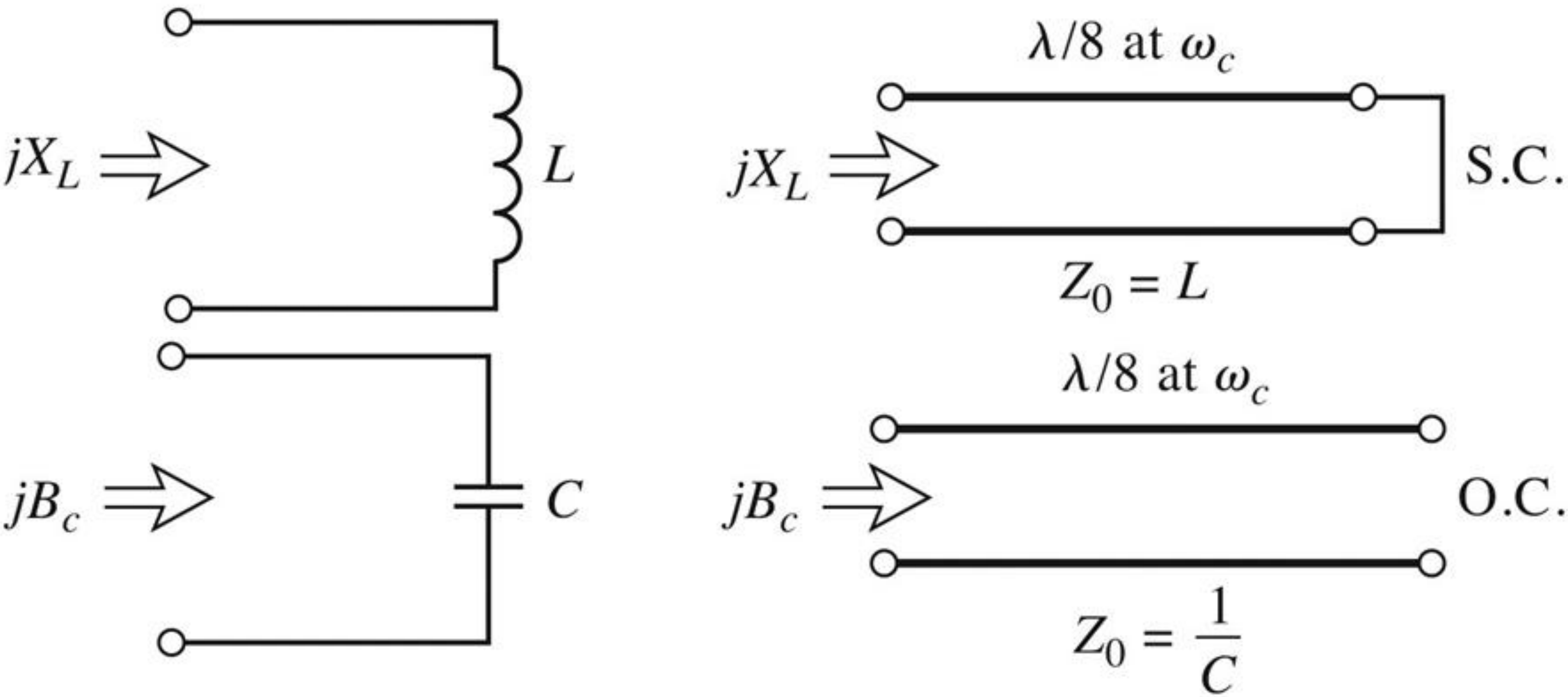
$$j \cdot B_C = j \cdot \Omega \cdot C = j \cdot C \cdot \tan \beta \cdot l$$

- The equivalent filter in Ω has a cutoff frequency at:

$$\Omega = 1 = \tan \beta \cdot l \quad \rightarrow \quad \beta \cdot l = \frac{\pi}{4} \quad \rightarrow \quad l = \frac{\lambda}{8}$$

Richards' Transformation

- allows implementation of the inductors and capacitors with lines **after** the transformation of the LPF prototype to the required type (LPF/HPF/BPF/BSF)



Richards' Transformation

- By choosing the open-circuited or short-circuited lines to be $\lambda/8$ at the desired cutoff frequency (ω_c) and the corresponding characteristic impedances (L/C from LPF prototype) we will obtain at frequencies around ω_c a behavior similar to that of the prototype filter.
  - At frequencies far from ω_c the behavior of the filter will no longer be identical to that of the prototype (in specific situations the correct behavior must be **verified**)
  - Frequency scaling is simplified: choosing the appropriate physical length of the line to have the electrical length $\lambda/8$ at the desired cutoff frequency
- All lines will have equal electrical lengths ($\lambda/8$) and thus comparable physical lengths, so the lines are called **commensurate** lines

Richards' Transformation

- At the frequency $\omega = 2 \cdot \omega_c$ the lines will be $\lambda/4$ long

$$l = \frac{\lambda}{4} \quad \Rightarrow \quad \beta \cdot l = \frac{\pi}{2} \quad \Rightarrow \quad \tan \beta \cdot l \to \infty$$

- an supplemental attenuation pole will occur at $2 \cdot \omega_c$ (LPF):
  - inductances (usually in series) $Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \to \infty$
  - capacitances (usually shunt) $Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l \to 0$

Richards' Transformation

- the periodicity of tan function implies the periodicity of the filter implemented with lines
  - the filter response will be repeated every $4\cdot\omega_c$

$$\tan(\alpha+\pi)=\tan\alpha$$

$$\beta\cdot l\Big|_{\omega=\omega_c}=\frac{\pi}{4}\quad\Rightarrow\quad\frac{\omega_c\cdot l}{v_p}=\frac{\pi}{4}\quad\Rightarrow\quad\pi=\frac{(4\cdot\omega_c)\cdot l}{v_p}$$

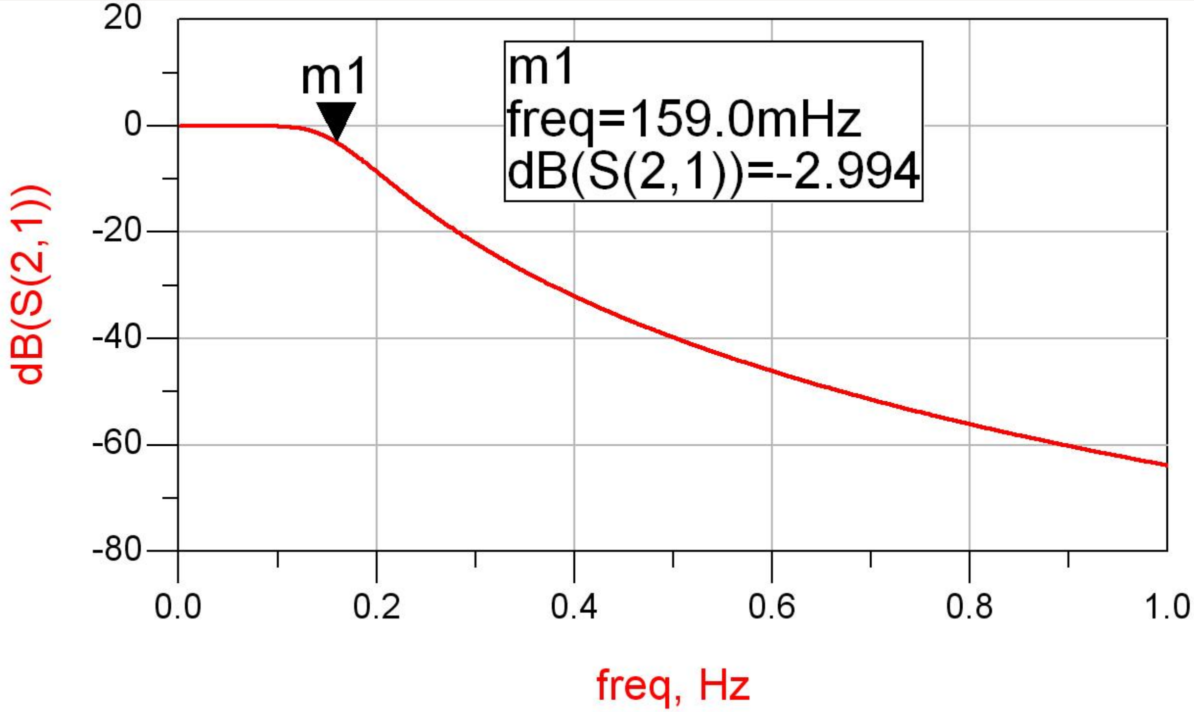
$$Z_{in}(\omega)=Z_{in}(\omega+4\cdot\omega_c)\quad\Rightarrow\quad P_{LR}(\omega)=P_{LR}(\omega+4\cdot\omega_c)$$

$$P_{LR}(4\cdot\omega_c)=P_{LR}(0)\qquad P_{LR}(3\cdot\omega_c)=P_{LR}(-\omega_c)\qquad P_{LR}(5\cdot\omega_c)=P_{LR}(\omega_c)$$

Example

- Low-pass filter 4th order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - g1 = 0.7654 = L1
  - g2 = 1.8478 = C2
  - g3 = 1.8478 = L3
  - g4 = 0.7654 = C4
  - g5 = 1 (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

LPF Prototype



Lumped elements

$$\omega_c = 2 \cdot \pi \cdot 4GHz = 2.5133 \cdot 10^{10} \, rad/s$$

g1 = 0.7654 = L1,

g2 = 1.8478 = C2,

g3 = 1.8478 = L3,

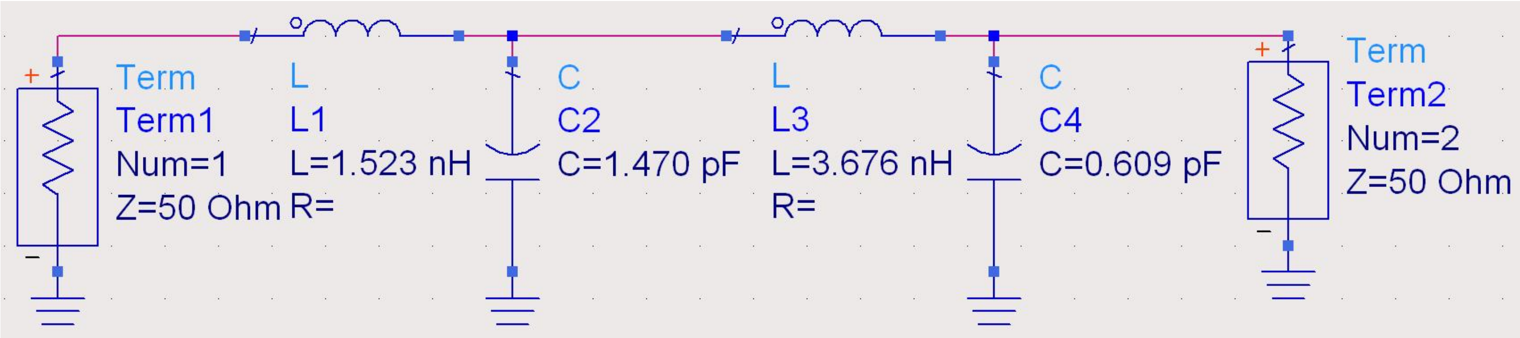
g4 = 0.7654 = C4,

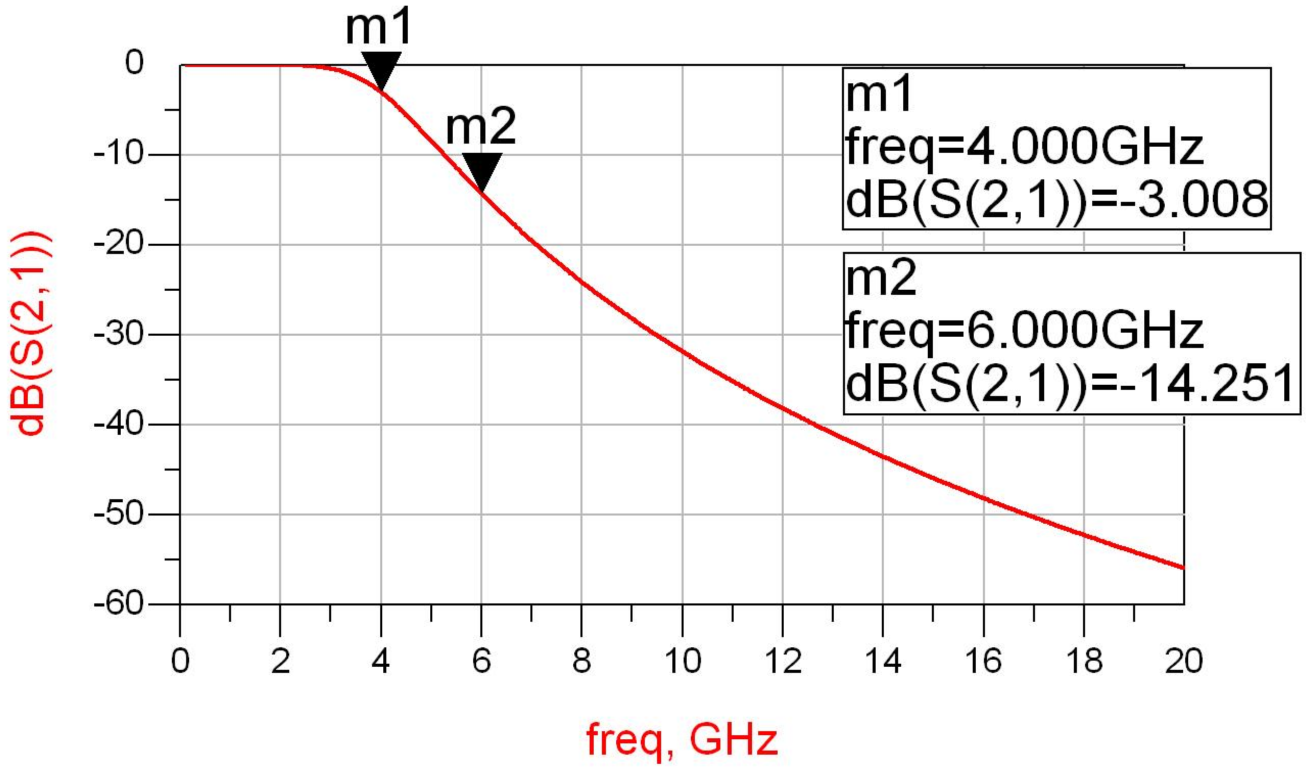
g5 = 1 = RL

$$L_1' = \frac{R_0 \cdot L_1}{\omega_c} = 1.523 nH \qquad C_2' = \frac{C_2}{R_0 \cdot \omega_c} = 1.470 \, pF$$

$$L_3' = \frac{R_0 \cdot L_3}{\omega_c} = 3.676 nH \qquad C_4' = \frac{C_4}{R_0 \cdot \omega_c} = 0.609 \, pF$$

Lumped elements – ADS





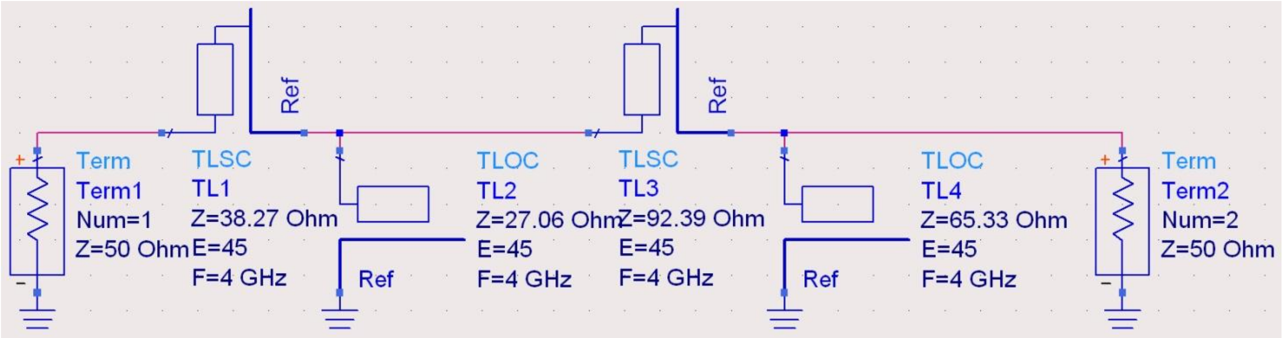
Richards' Transformation

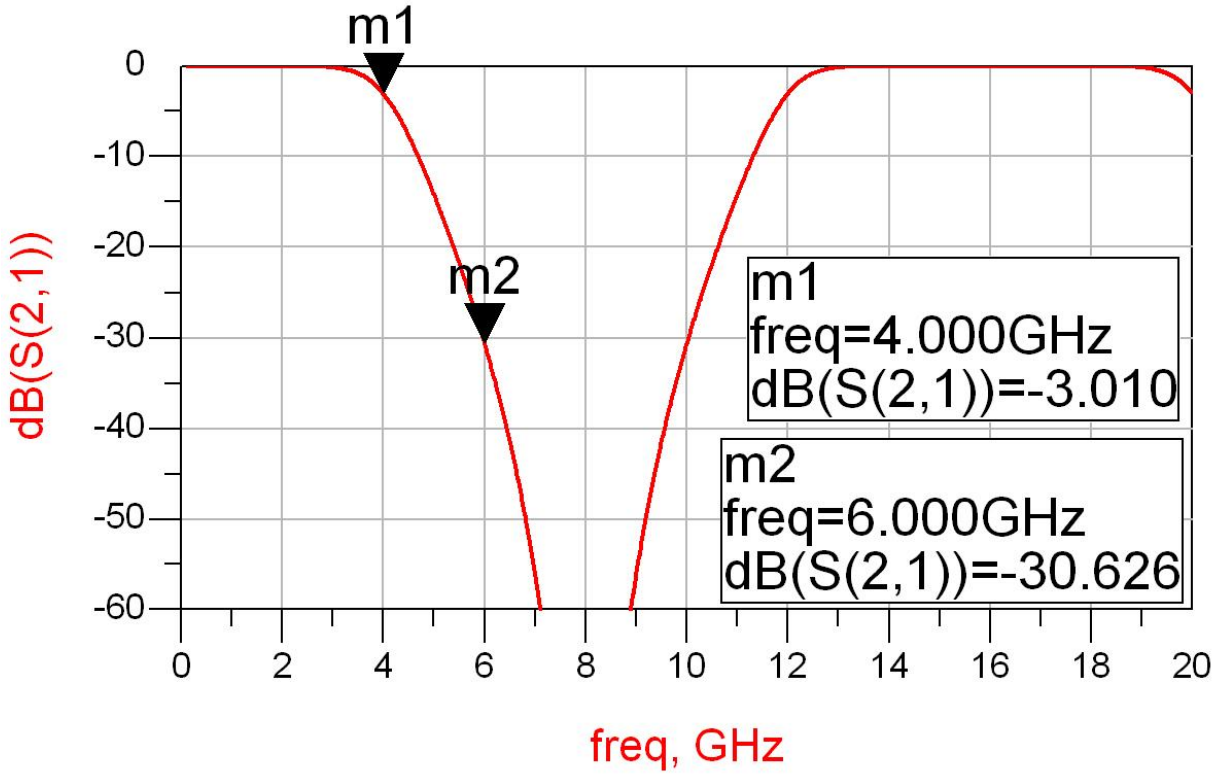
- LPF Prototype parameters:
  - g1 = 0.7654 = L1
  - g2 = 1.8478 = C2
  - g3 = 1.8478 = L3
  - g4 = 0.7654 = C4
- Normalized line impedances
  - z1 = 0.7654 = series / short circuit
  - z2 = 1 / 1.8478 = 0.5412 = shunt / open circuit
  - z3 = 1.8478 = series / short circuit
  - z4 = 1/ 0.7654 = 1.3065 = shunt / open circuit
- Impedance scaling by multiplying with Zo = 50Ω
- All lines must have the length equal to λ/8 (electrical length E = 45°) at 4GHz

$$Z_0 \leftrightarrow \frac{1}{C}$$

$$Z_0 \leftrightarrow L$$

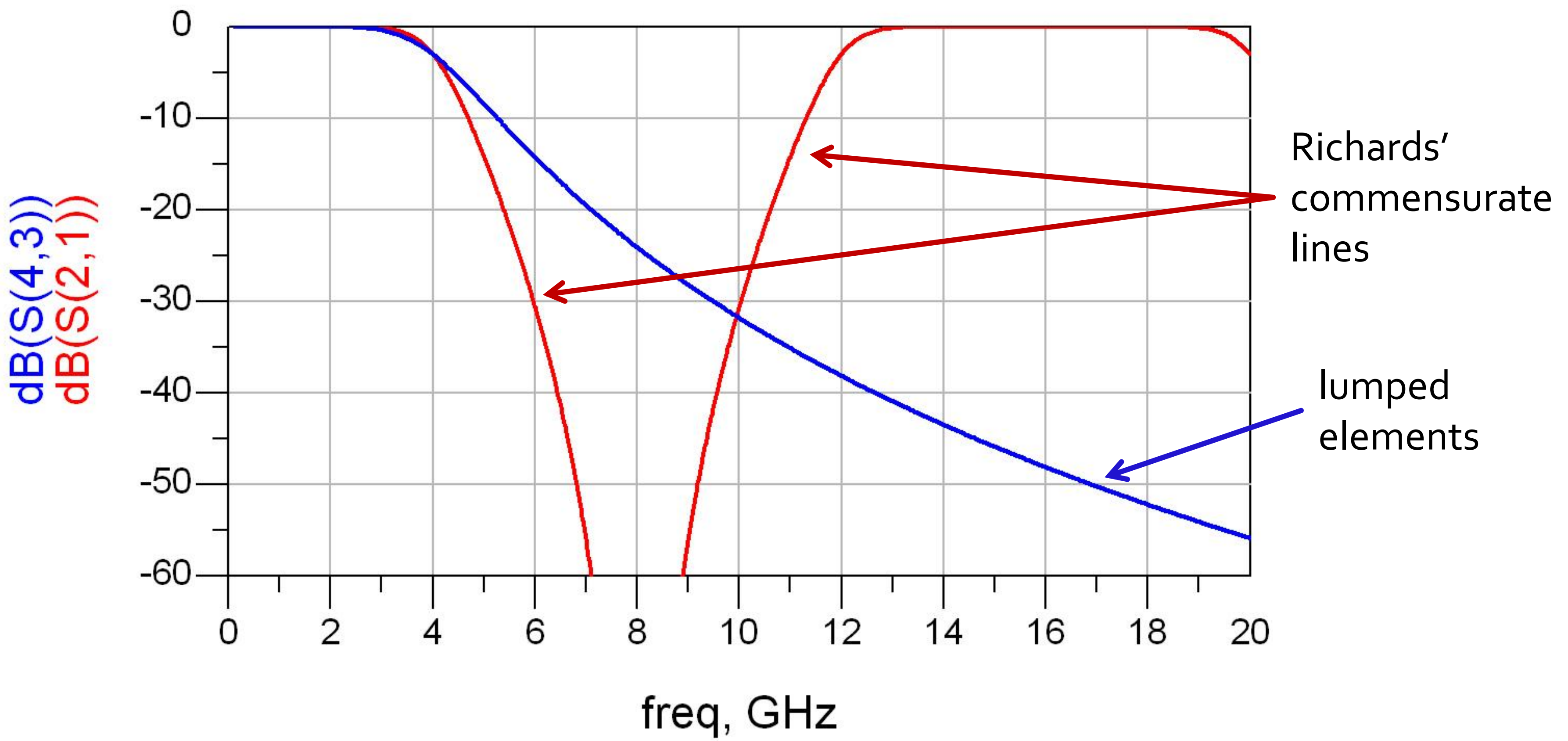
Richards' Transformation – ADS





Richards' Transformation

- Filters implemented with Richards' Transformation
  - beneficiate from the supplemental pole at $2 \cdot \omega_c$
  - have the major disadvantage of frequency periodicity, a supplemental non-periodic LPF must be inserted if needed



Equal-ripple prototype

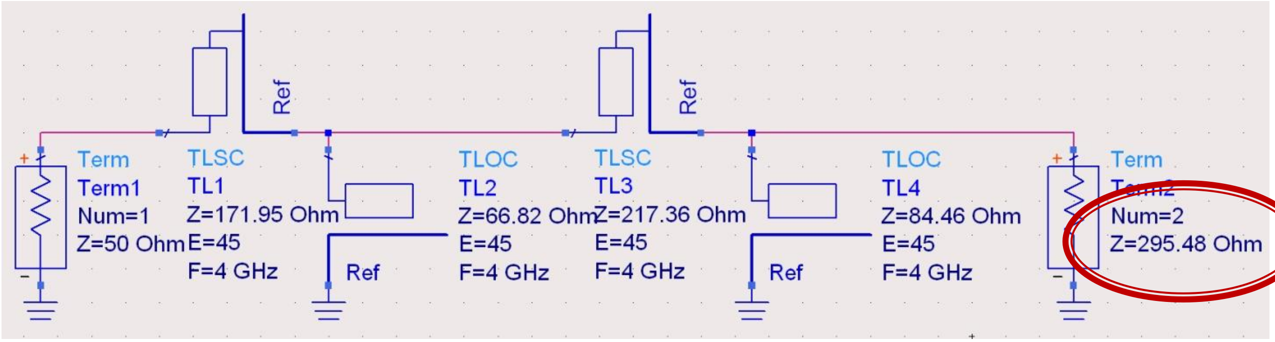
- For even N order of the filter (N = 2, 4, 6, 8 …) equal-ripple filters **must** closed by a non-standard load impedance $\mathbf{g_{N+1} \neq 1}$
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial …) to $\mathbf{g_L = 1}$

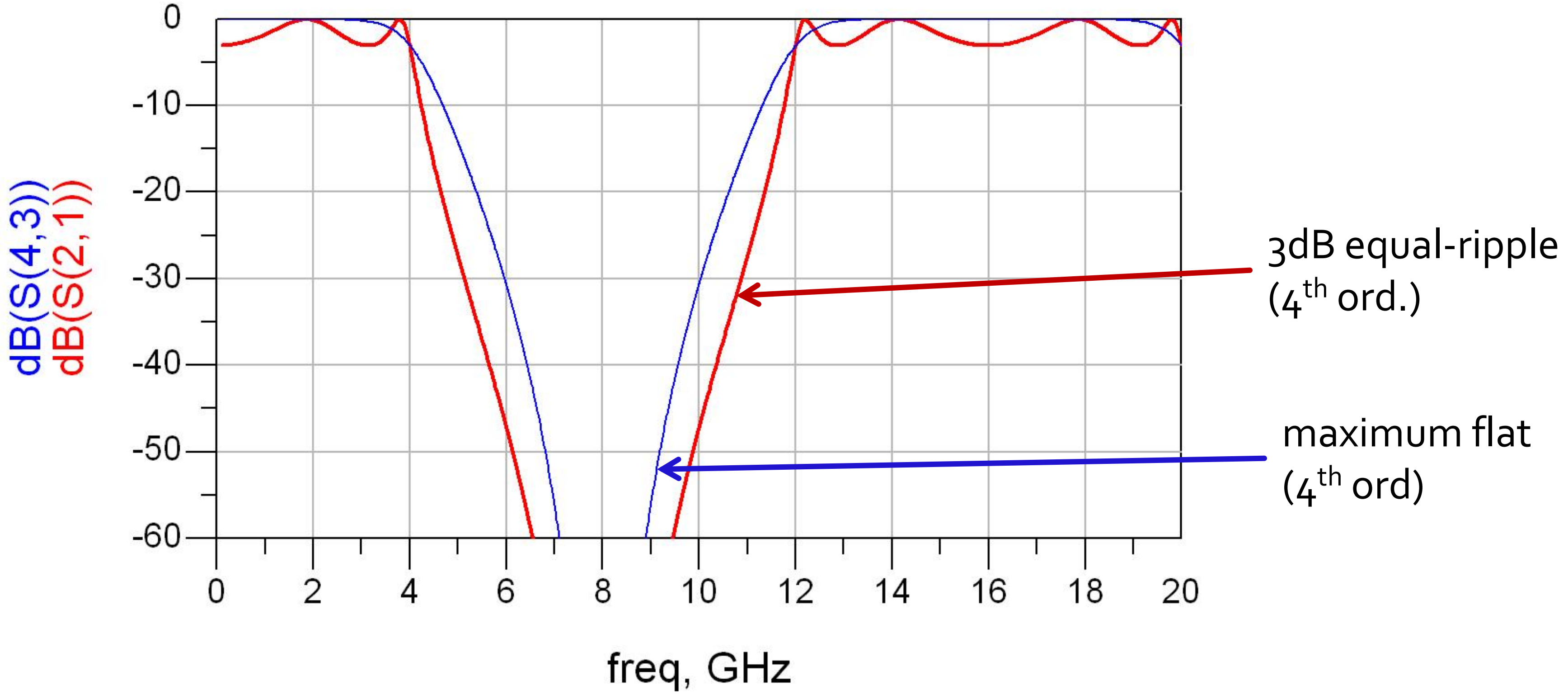
$$g_{N+1} \neq 1 \rightarrow R \neq R_0 \quad (50\Omega)$$

Observation: even order equal-ripple

- Same filter, 3dB equal-ripple
- 3dB equal-ripple tables or formulas:
  - $g1 = 3.4389 = L1$
  - $g2 = 0.7483 = C2$
  - $g3 = 4.3471 = L3$
  - $g4 = 0.5920 = C4$
  - $g5 = 5.8095 = R_L$
- Line impedances
  - $Z1 = 3.4389 \cdot 50\Omega = 171.945\Omega$ = series / short circuit
  - $Z2 = 50\Omega / 0.7483 = 66.818\Omega$ = shunt / open circuit
  - $Z3 = 4.3471 \cdot 50\Omega = 217.355\Omega$ = series / short circuit
  - $Z4 = 50\Omega / 0.5920 = 84.459\Omega$ = shunt / open circuit
  - $RL = 5.8095 \cdot 50\Omega = 295.475\Omega$ = load

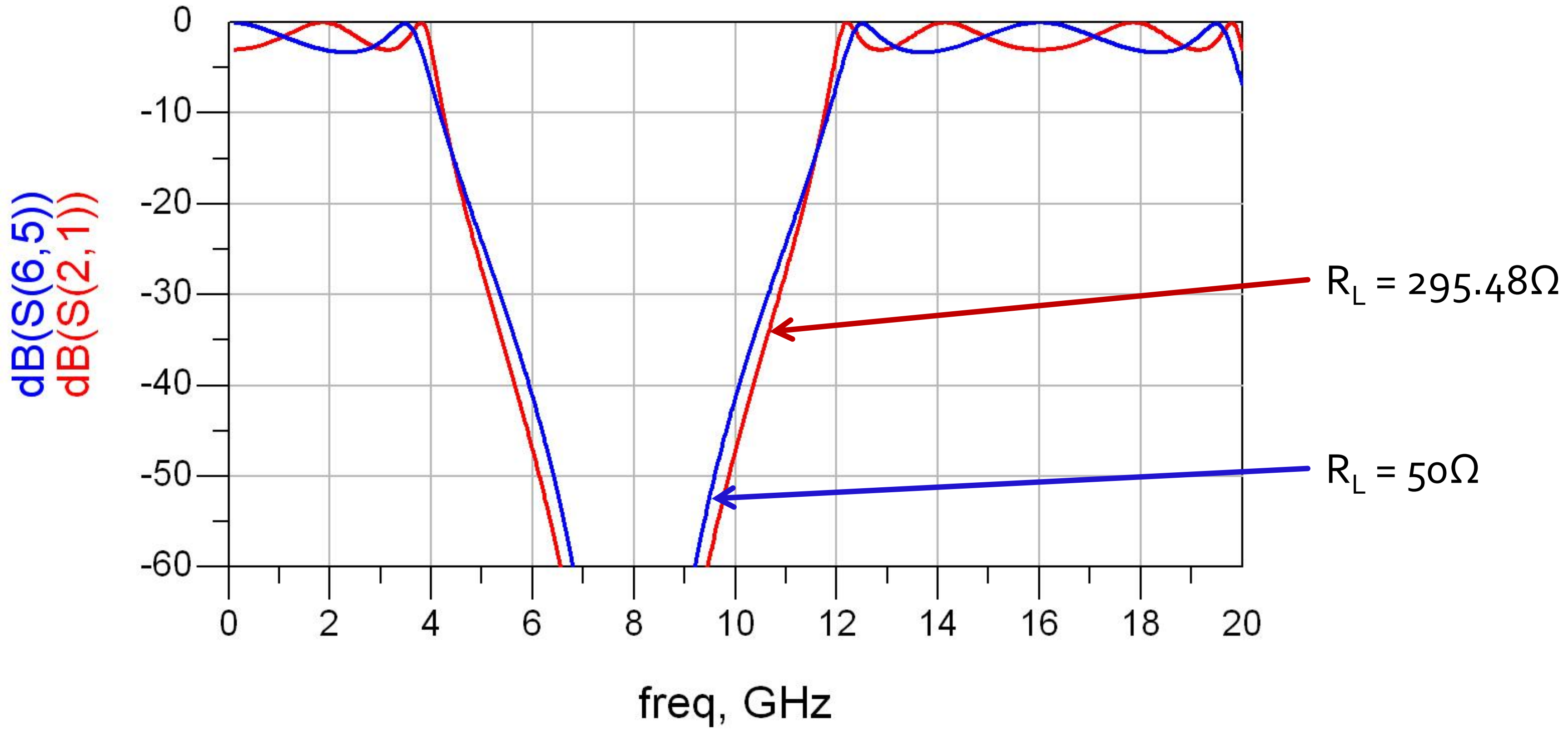
Even order equal-ripple – ADS





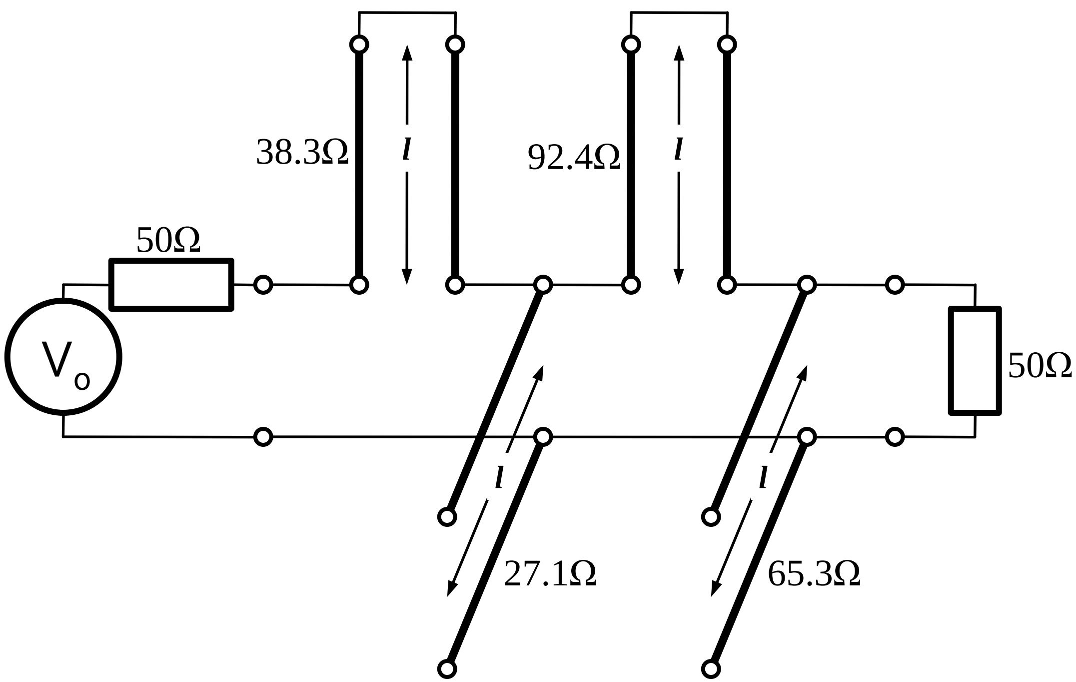
Observation: even order equal-ripple

- Even order equal-ripple filters need output matching towards 50Ω for precise results. Example:



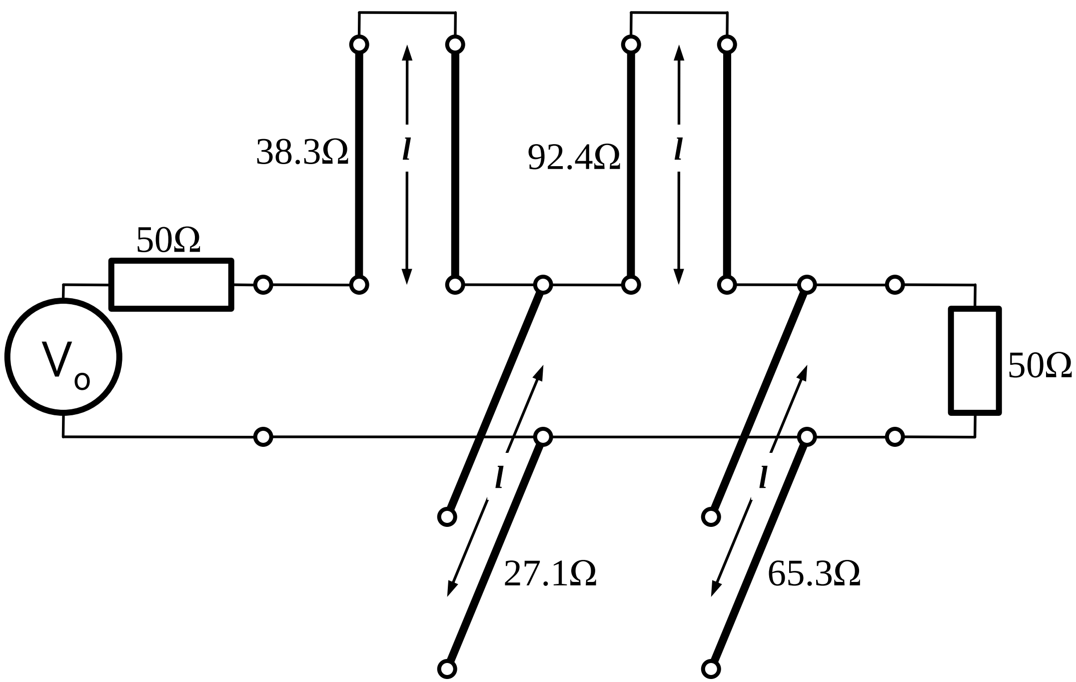
Kuroda's Identities

- Filters implemented with the Richards' transformation have certain disadvantages in terms of practical use
- Kuroda's Identities/Transformations can eliminate some of these disadvantages
- We use additional line sections to obtain systems that are easier to implement in practice
- The additional line sections are called unit elements and have lengths of λ / 8 at the desired cutoff frequency (ω_c) thus being commensurate with the stubs implementing the inductors and capacitors.



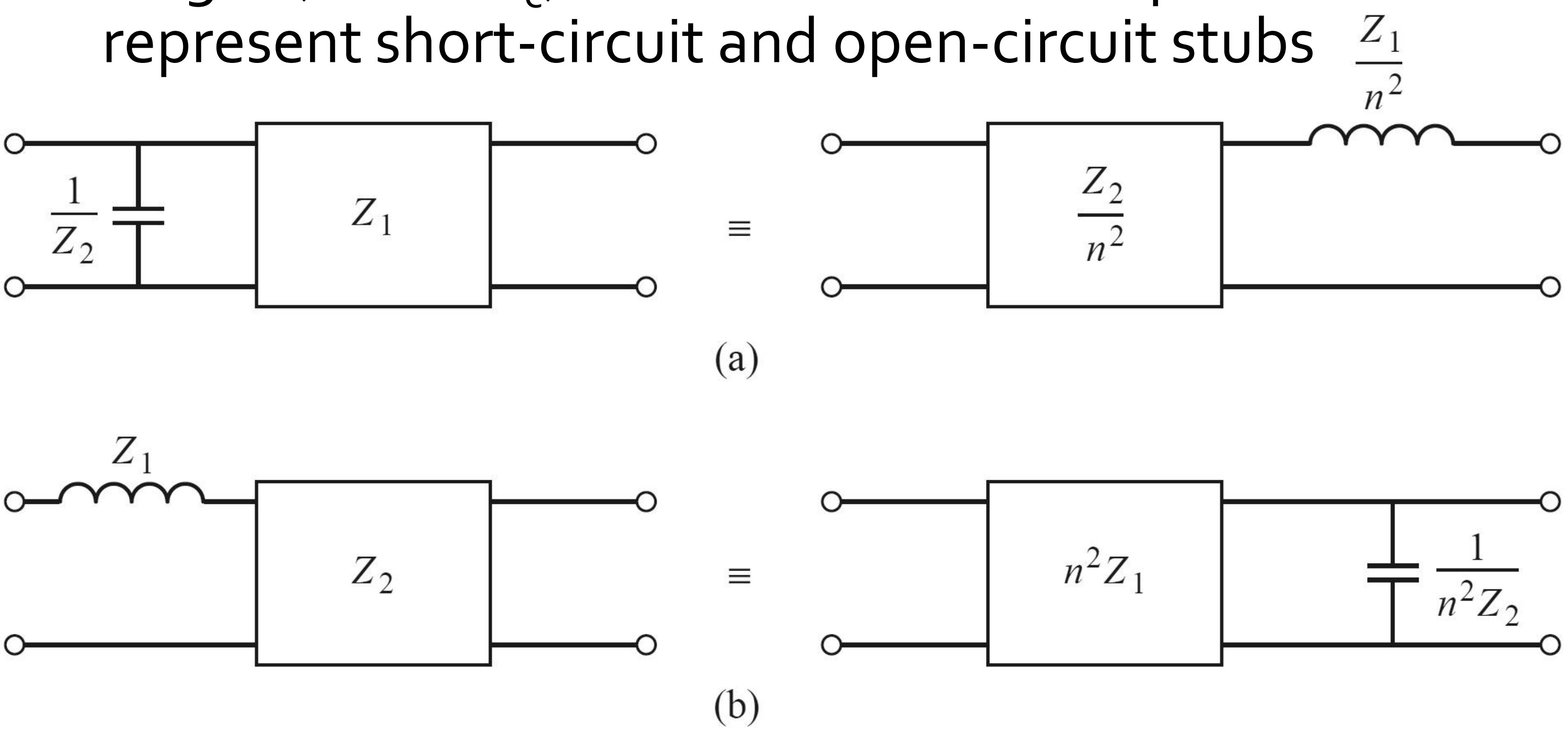
Kuroda's Identities

- Kuroda's Identities perform any of the following operations:
  - Physically separate transmission line stubs
  - Transform series stubs into shunt stubs, or vice versa
  - Change impractical characteristic impedances into more realizable values (~50Ω)



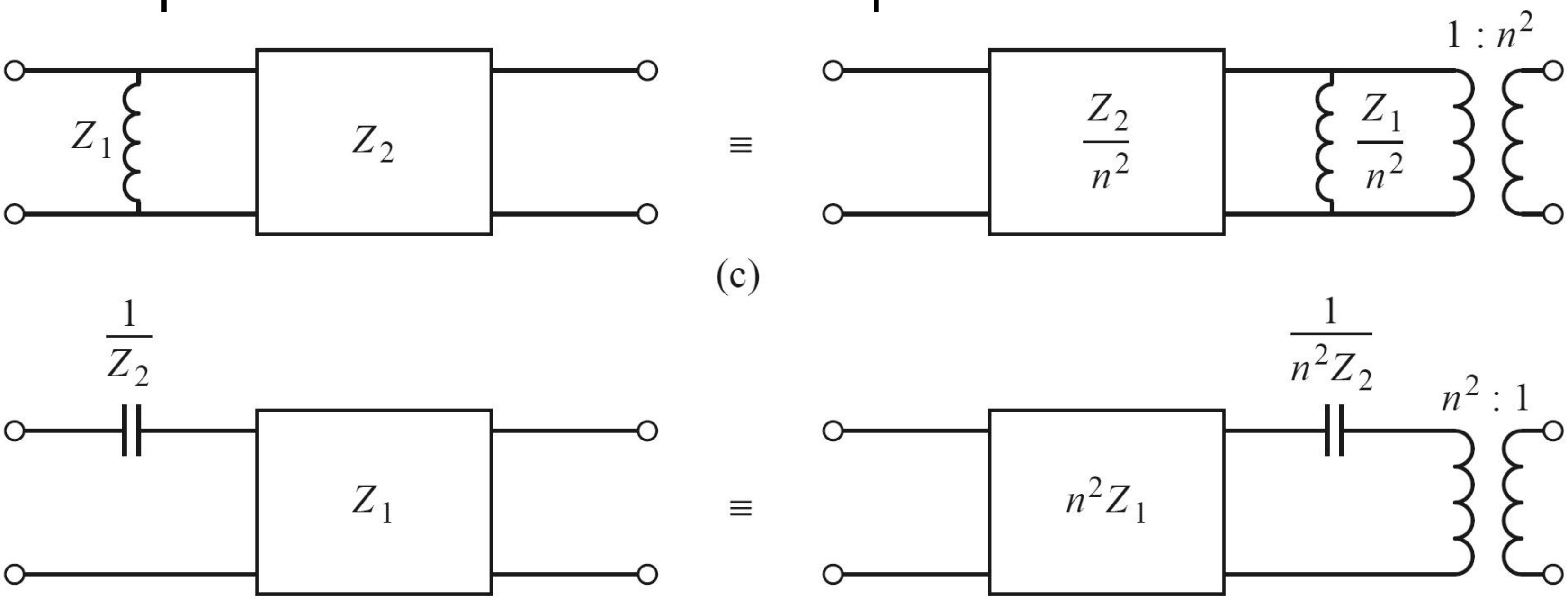
Kuroda's Identities

- 4 circuit equivalents (a,b)
  - each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c). The inductors and capacitors represent short-circuit and open-circuit stubs



Kuroda's Identities

- 4 circuit equivalents (c,d)
  - each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c). The inductors and capacitors represent short-circuit and open-circuit stubs



(d)

Kuroda's Identities

- In all Kuroda's Identities:
  - n:

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

  - The inductors and capacitors represent short-circuit and open-circuit stubs resulted from Richards' transformation ($\lambda/8$ at ω_c).
  - Each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c).

First Kuroda's Identity

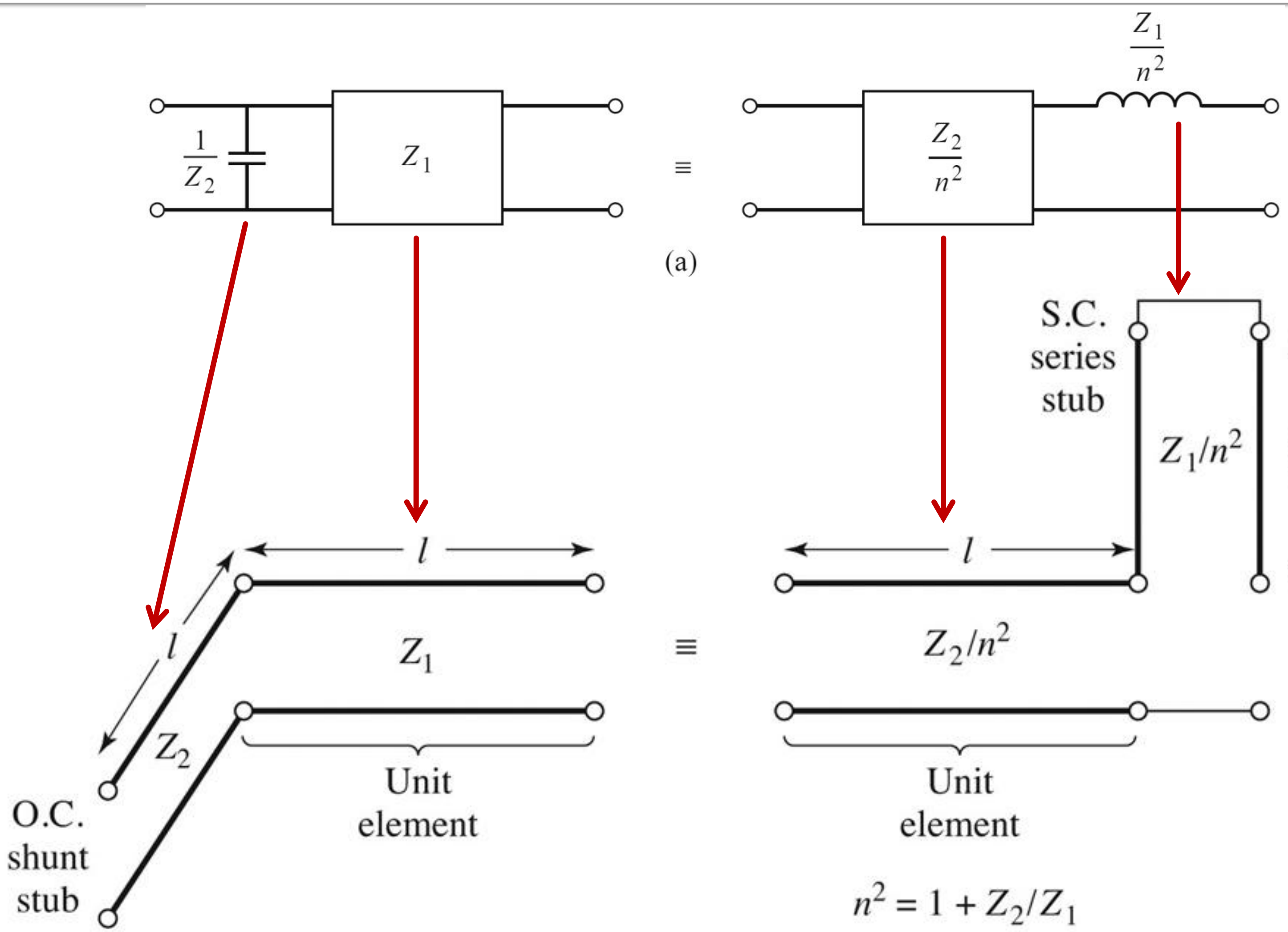
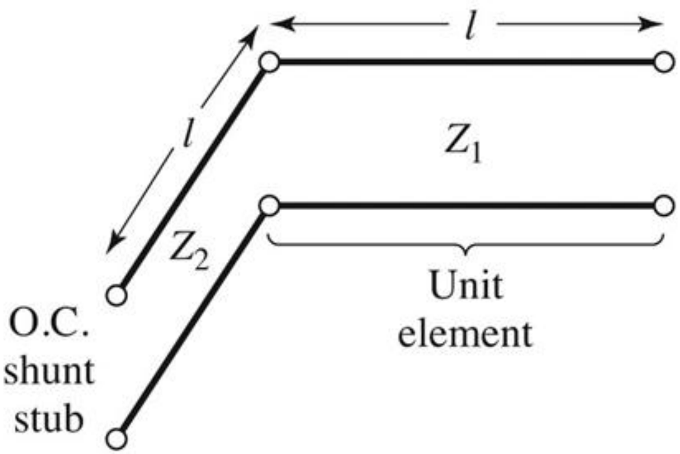


Figure 8.35

First Kuroda's Identity – Proof



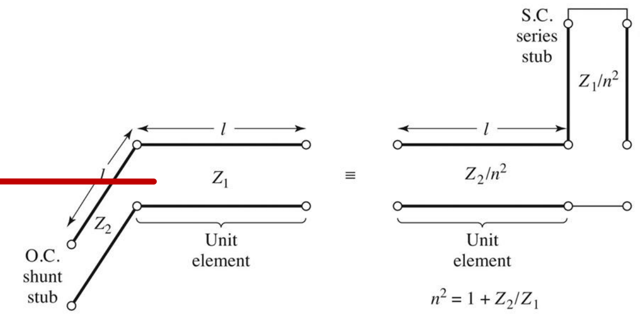
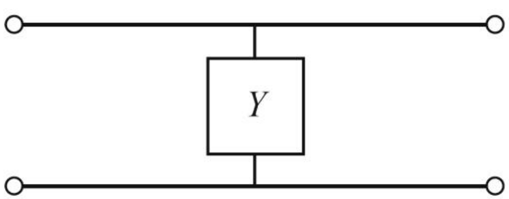


Figure 8.35

- ABCD matrix, L5



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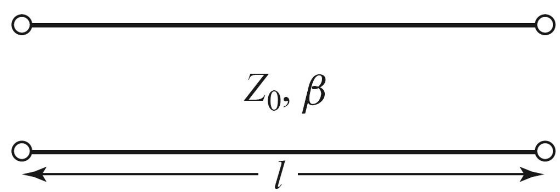
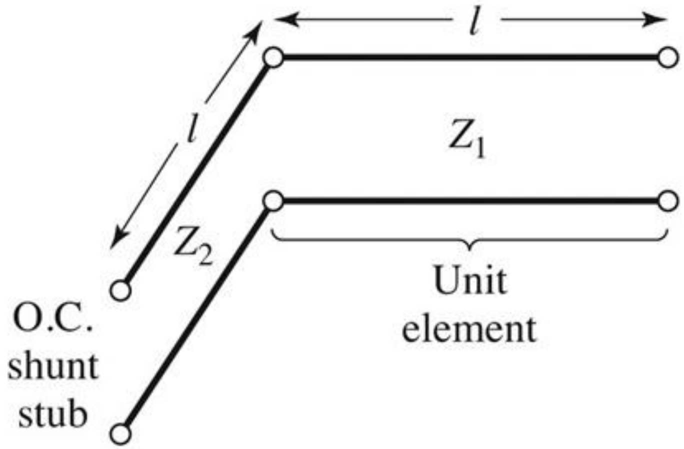


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta \cdot l & j \cdot Z_0 \cdot \sin\beta \cdot l \\ j \cdot Y_0 \cdot \sin\beta \cdot l & \cos\beta \cdot l \end{bmatrix}$$

First Kuroda's Identity – Proof



$$\Omega = \tan \beta \cdot l$$

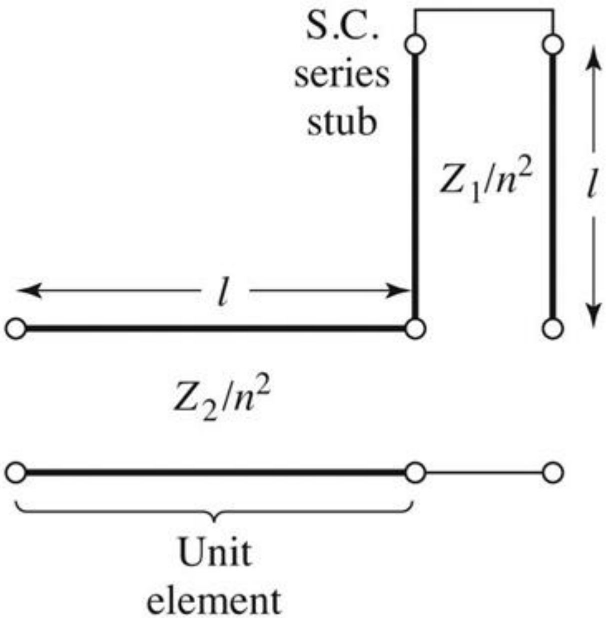
$$\cos \beta \cdot l = \frac{1}{\sqrt{1+\Omega^2}} \qquad \sin \beta \cdot l = \frac{\Omega}{\sqrt{1+\Omega^2}}$$

$$Z_{in,oc} = -j \cdot Z_2 \cdot \cot \beta \cdot l = -j \cdot \frac{Z_2}{\Omega}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{j \cdot \Omega}{Z_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{1+\Omega^2}} & j \cdot Z_1 \cdot \frac{\Omega}{\sqrt{1+\Omega^2}} \\ j \cdot \frac{1}{Z_1} \cdot \frac{\Omega}{\sqrt{1+\Omega^2}} & \frac{1}{\sqrt{1+\Omega^2}} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & 0 \\ \frac{j \cdot \Omega}{Z_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ \frac{j \cdot \Omega}{Z_1} & 1 \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ j \cdot \Omega \cdot \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

First Kuroda's Identity – Proof



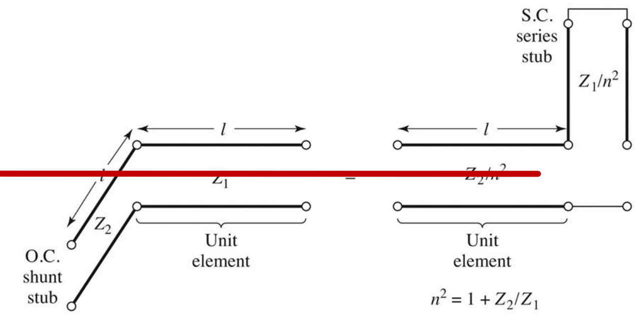
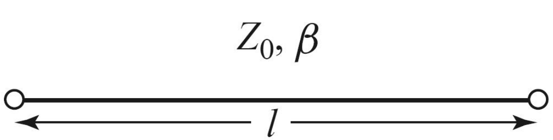


Figure 8.35



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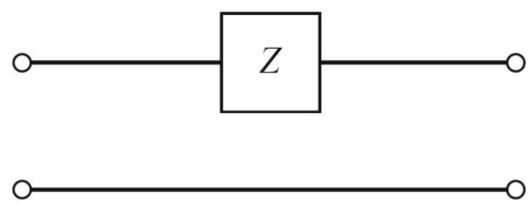
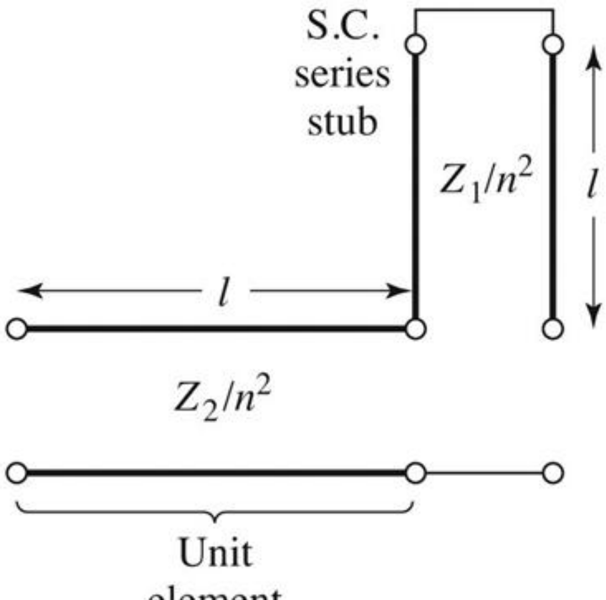


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\cdot l & j\cdot Z_0\cdot\sin\beta\cdot l \\ j\cdot Y_0\cdot\sin\beta\cdot l & \cos\beta\cdot l \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

First Kuroda's Identity – Proof



$$\Omega = \tan\beta\cdot l$$

$$\cos\beta\cdot l = \frac{1}{\sqrt{1+\Omega^2}} \qquad \sin\beta\cdot l = \frac{\Omega}{\sqrt{1+\Omega^2}}$$

$$Z_{in,sc} = j\cdot\left(\frac{Z_1}{n^2}\right)\cdot\tan\beta\cdot l = \frac{j\cdot\Omega\cdot Z_1}{n^2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1+\Omega^2}} & j\cdot\frac{Z_2}{n^2}\cdot\frac{\Omega}{\sqrt{1+\Omega^2}} \\ j\cdot\frac{n^2}{Z_2}\cdot\frac{\Omega}{\sqrt{1+\Omega^2}} & \frac{1}{\sqrt{1+\Omega^2}} \end{bmatrix}\cdot\begin{bmatrix} 1 & \frac{j\cdot\Omega\cdot Z_1}{n^2} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}}\cdot\begin{bmatrix} 1 & j\cdot\Omega\cdot\frac{Z_2}{n^2} \\ \frac{j\cdot\Omega\cdot n^2}{Z_2} & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & j\cdot\Omega\cdot\frac{Z_1}{n^2} \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}}\cdot\begin{bmatrix} 1 & j\cdot\frac{\Omega}{n^2}\cdot(Z_1+Z_2) \\ \frac{j\cdot\Omega\cdot n^2}{Z_2} & 1-\Omega^2\cdot\frac{Z_1}{Z_2} \end{bmatrix}$$

First Kuroda's Identity – Proof

- First circuit

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ j \cdot \Omega \cdot \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

- Second circuit

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \frac{\Omega}{n^2} \cdot (Z_1 + Z_2) \\ \frac{j \cdot \Omega \cdot n^2}{Z_2} & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

- Results are identical if we choose

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

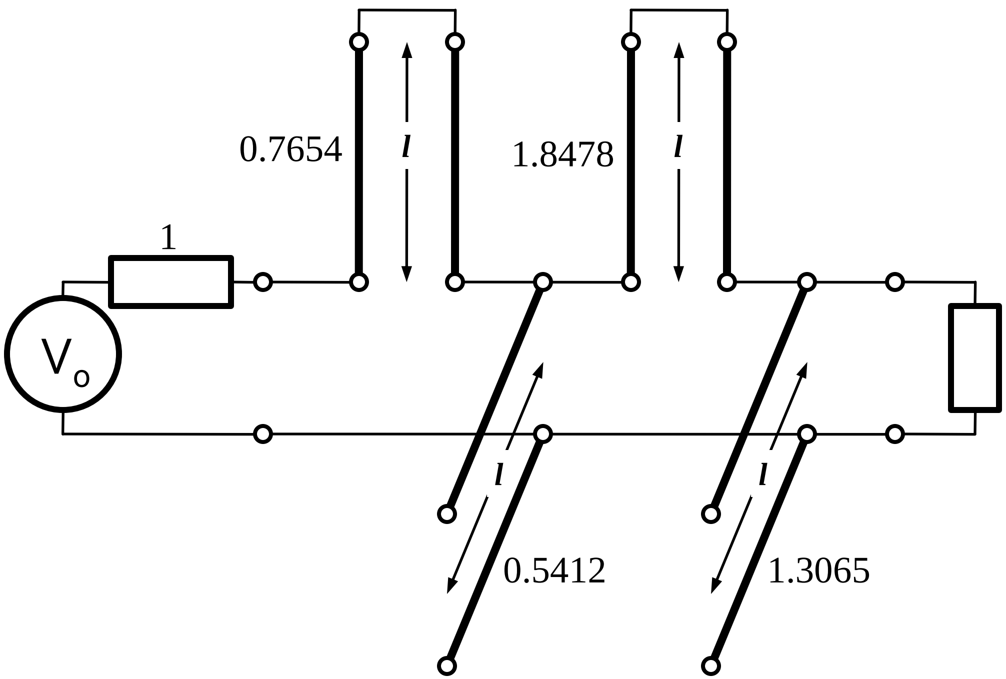
- The other 3 identities can be proved in the same way

(Same) Example

- Low-pass filter 4th order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g_1 = 0.7654 = L_1$
  - $g_2 = 1.8478 = C_2$
  - $g_3 = 1.8478 = L_3$
  - $g_4 = 0.7654 = C_4$
  - $g_5 = 1$ (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

Example

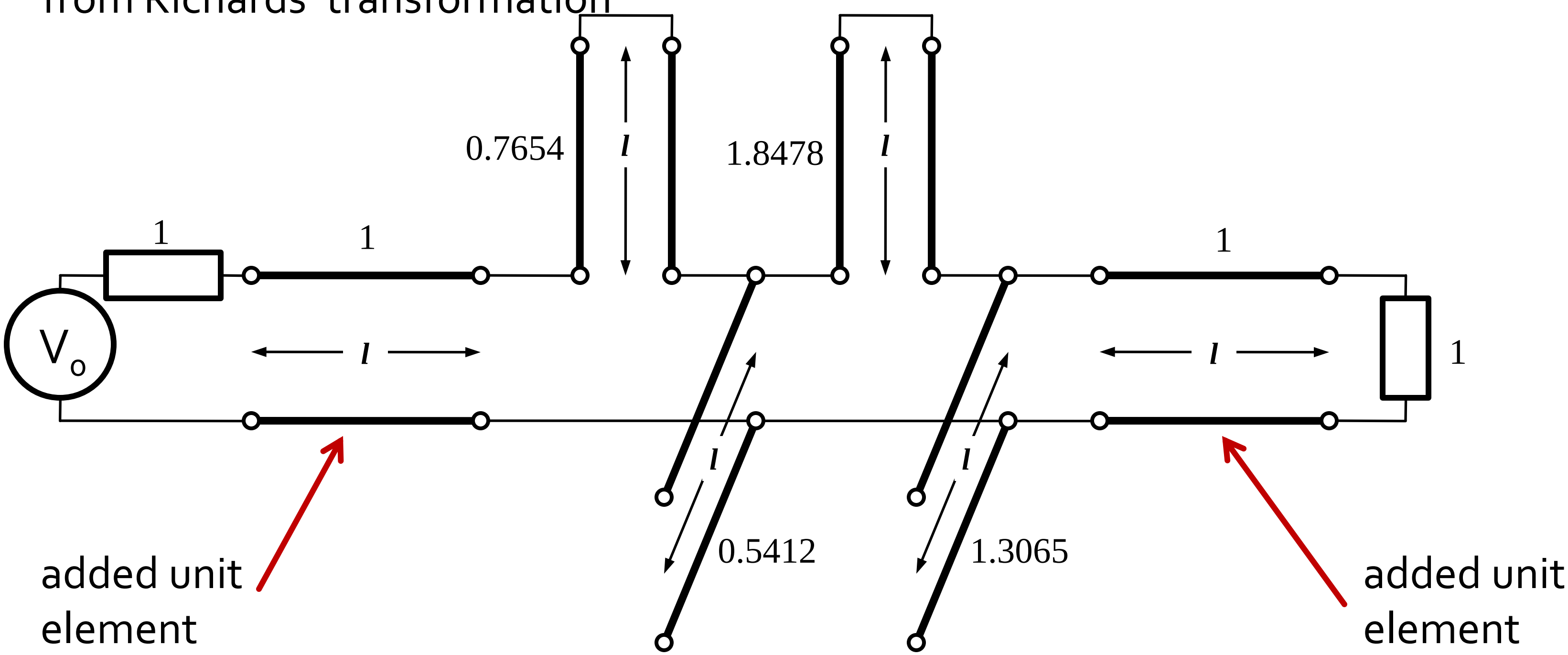
- Apply Richards's transformation



- Problems:
  - the series stubs would be **very difficult** to implement in microstrip line form
  - in microstrip technology it is preferable to have open-circuit stubs (short-circuit requires a **via-hole** to the ground plane)
  - the 4 stubs are physically connected at the same point, an implementation that eliminates/reduces the **coupling** between these lines is impossible
  - not the case here, but sometimes the normalized impedances are much different from 1. Most circuit technologies are designed for 50Ω lines

Example

- In all 4 Kuroda's Identities we **always** have a circuit with a series line section (not present in initial circuit):
  - we **add** unit elements (z = 1, l = λ/8) at the ends of the filter (these redundant elements do not affect filter performance since they are matched to z = 1, both source and load)
  - we **apply** one of the Kuroda's Identities at both ends and **continue** (add unit …)
  - we can **stop** the procedure when we have a series line section between all the stubs from Richards' transformation

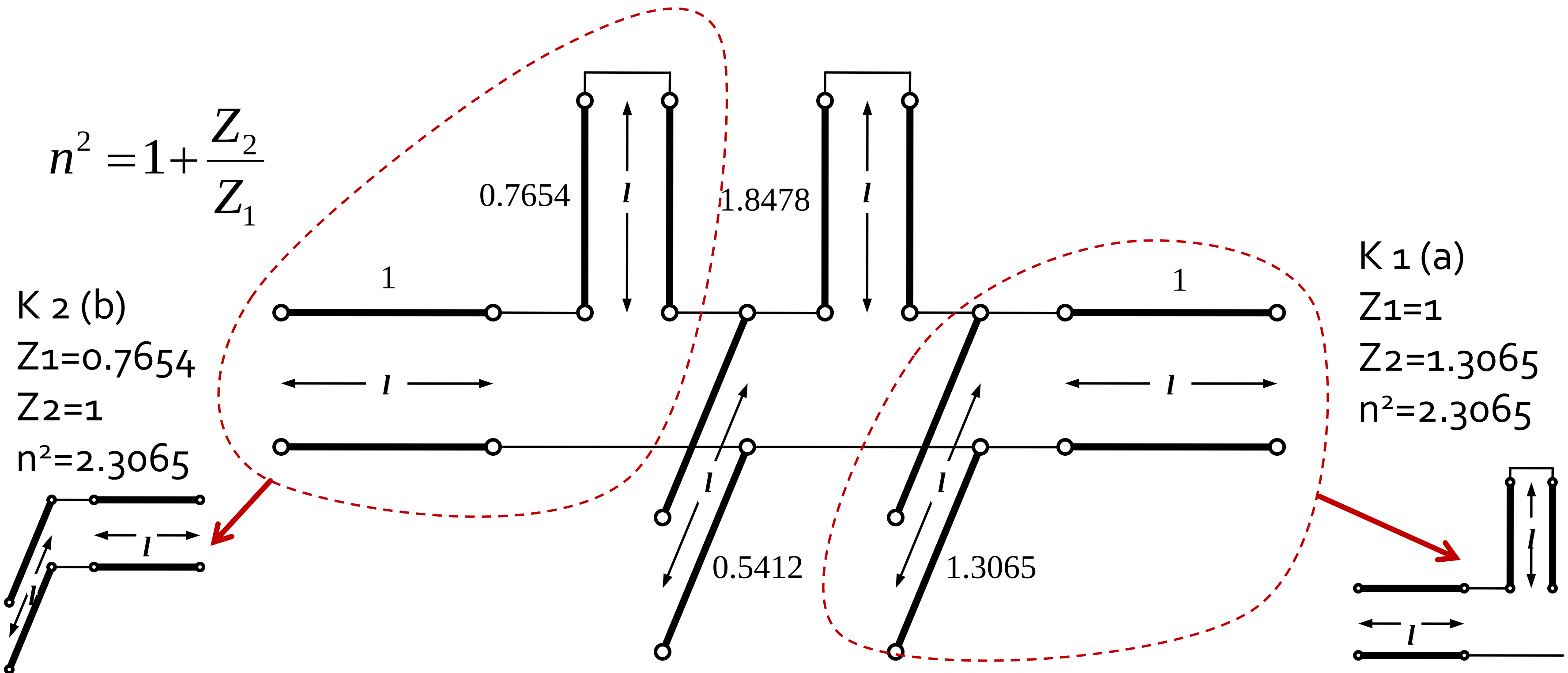


Example

- Apply:
  - Kuroda 2 (L,Z known → C,Z) on the left side
  - Kuroda 1 (C,Z known → L,Z) on the right side

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

K 2 (b)
Z1=0.7654
Z2=1
n^2=2.3065

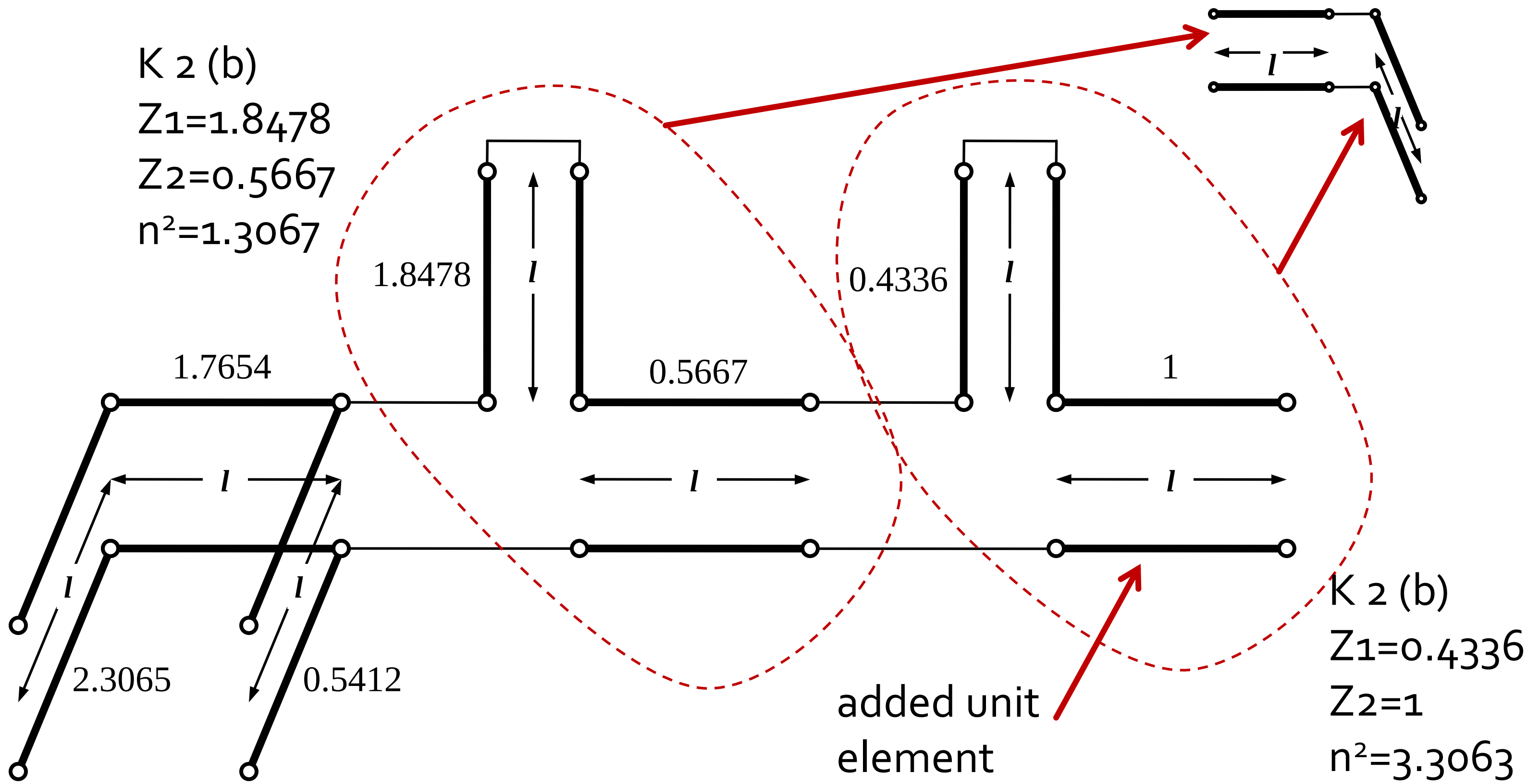


K 1 (a)
Z1=1
Z2=1.3065
n^2=2.3065

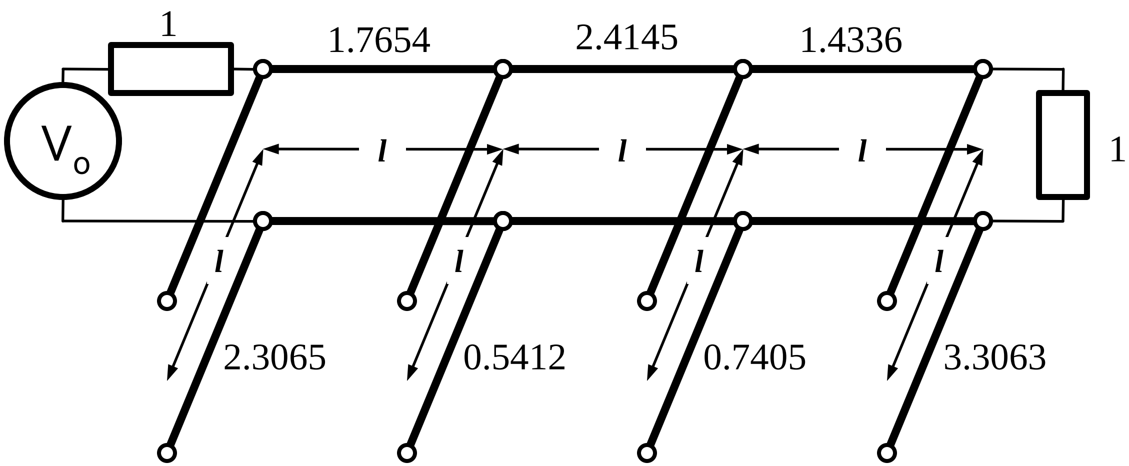
Example

- We add another unit element on the right side and apply Kuroda 2 twice

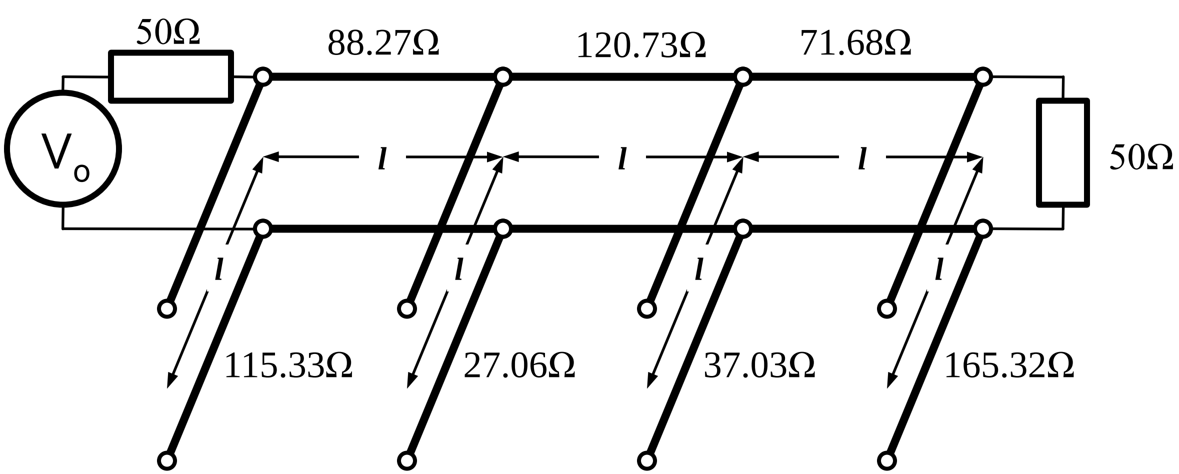
$$n^2 = 1 + \frac{Z_2}{Z_1}$$



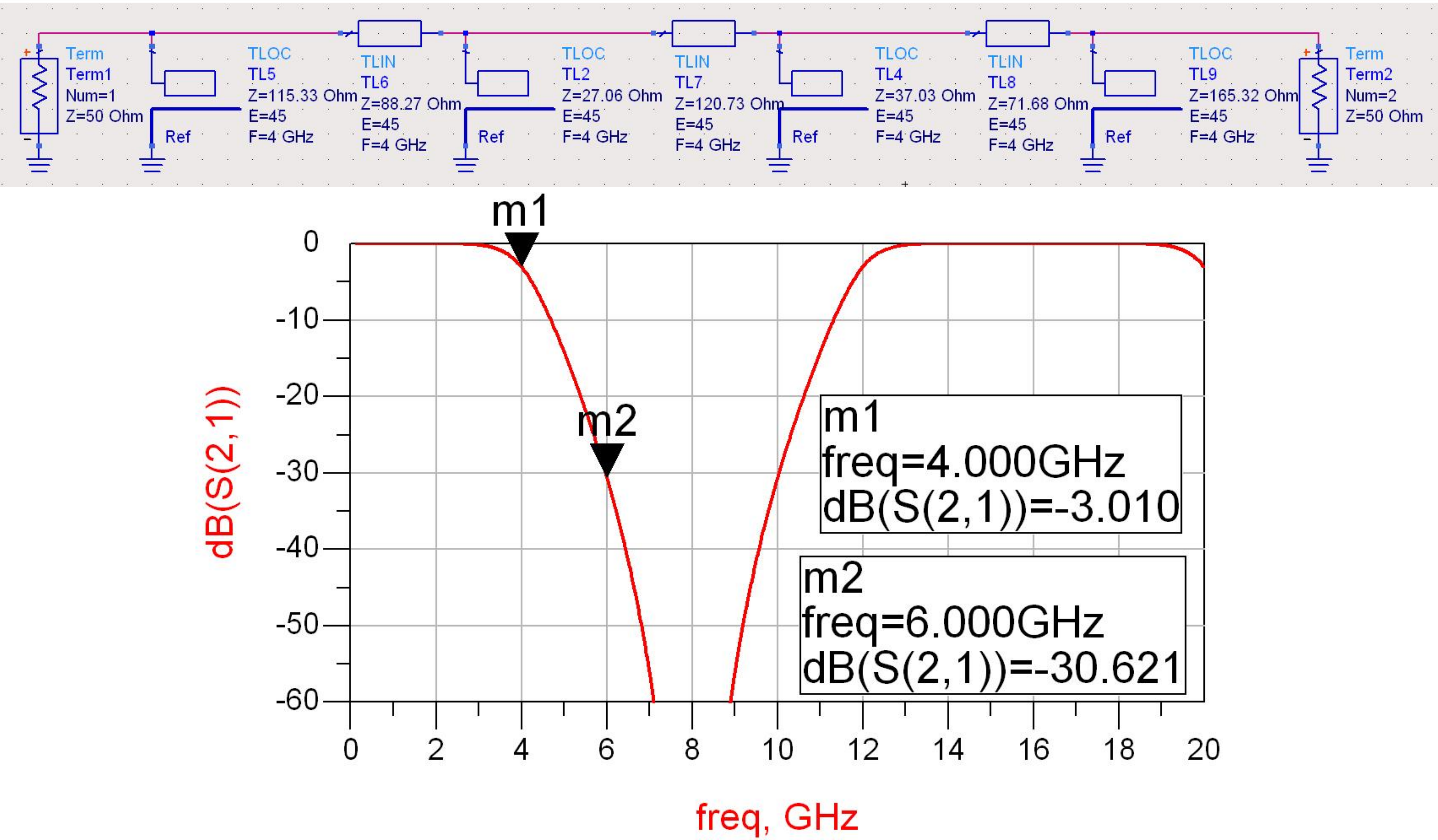
Example



- Impedance scaling (multiply by 50Ω)



Kuroda's Identities – ADS



Examples

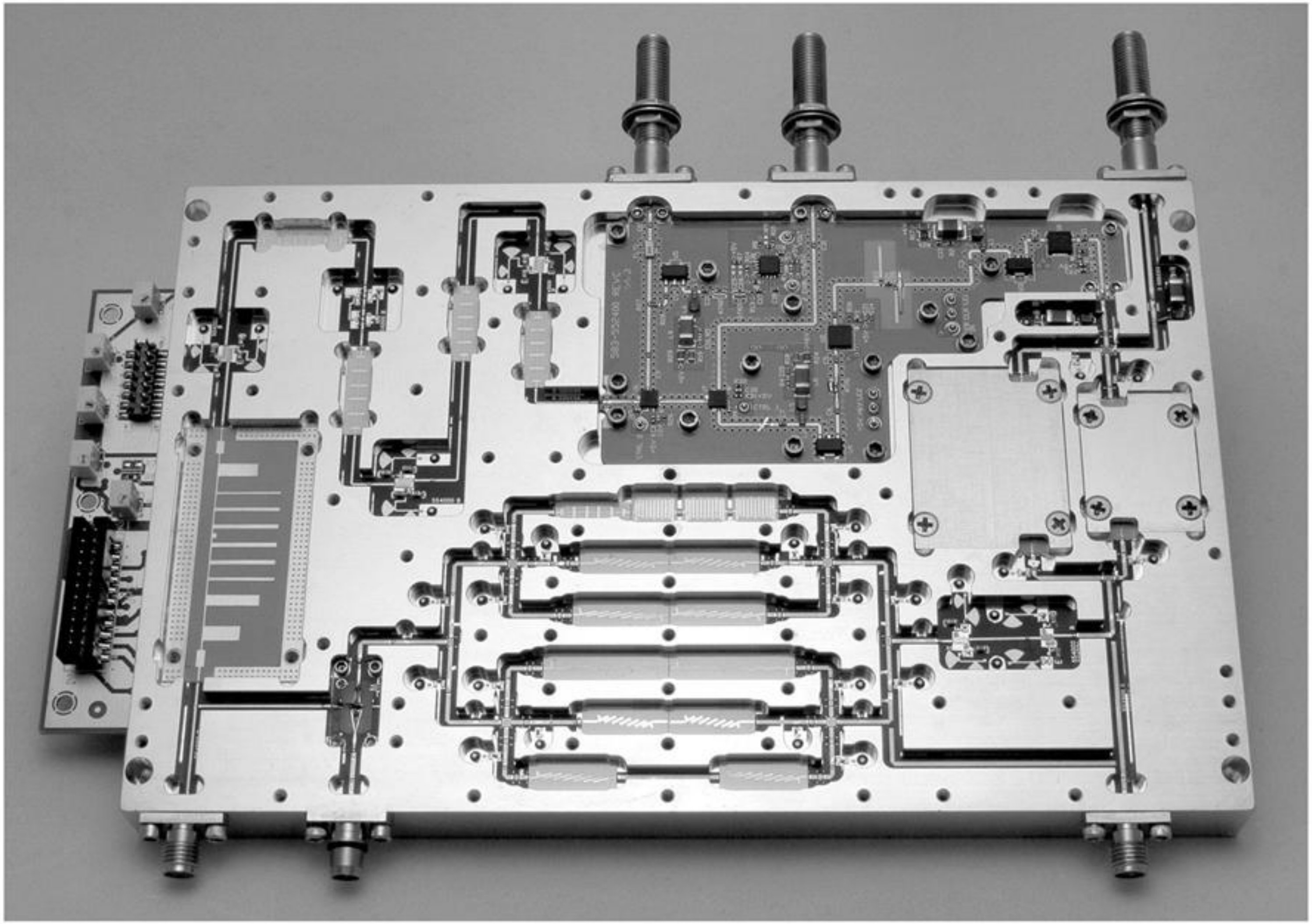


Figure 8.55
Courtesy of LNX Corporation, Salem, N.H.

Examples

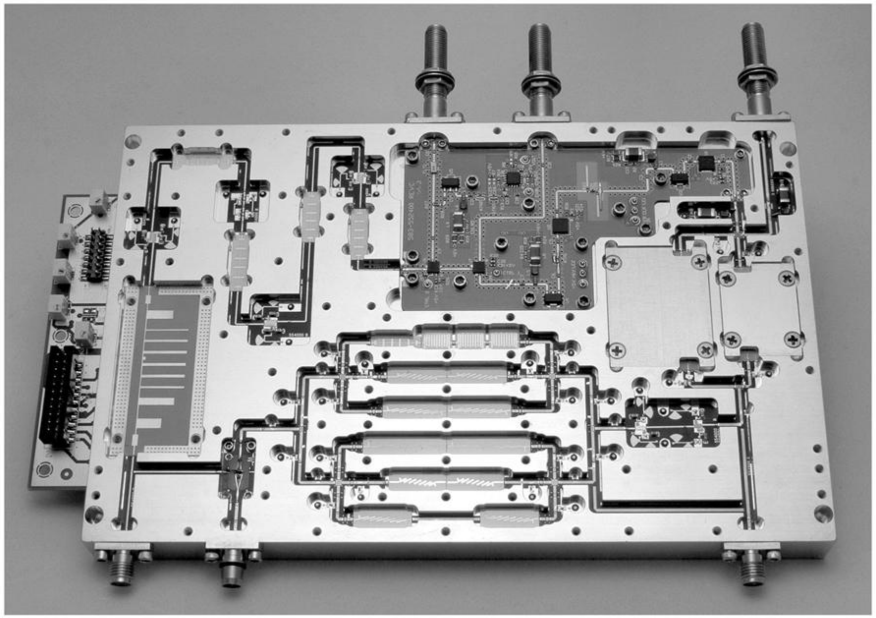
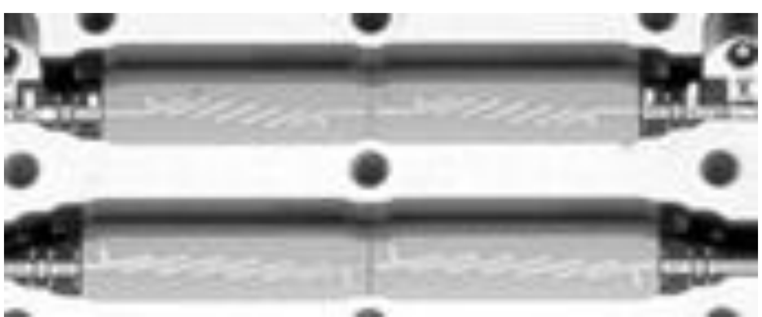
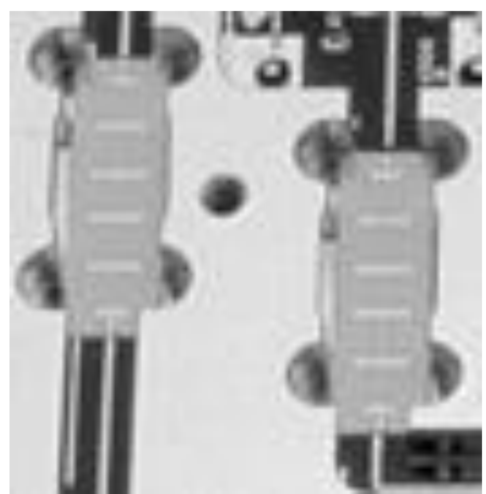
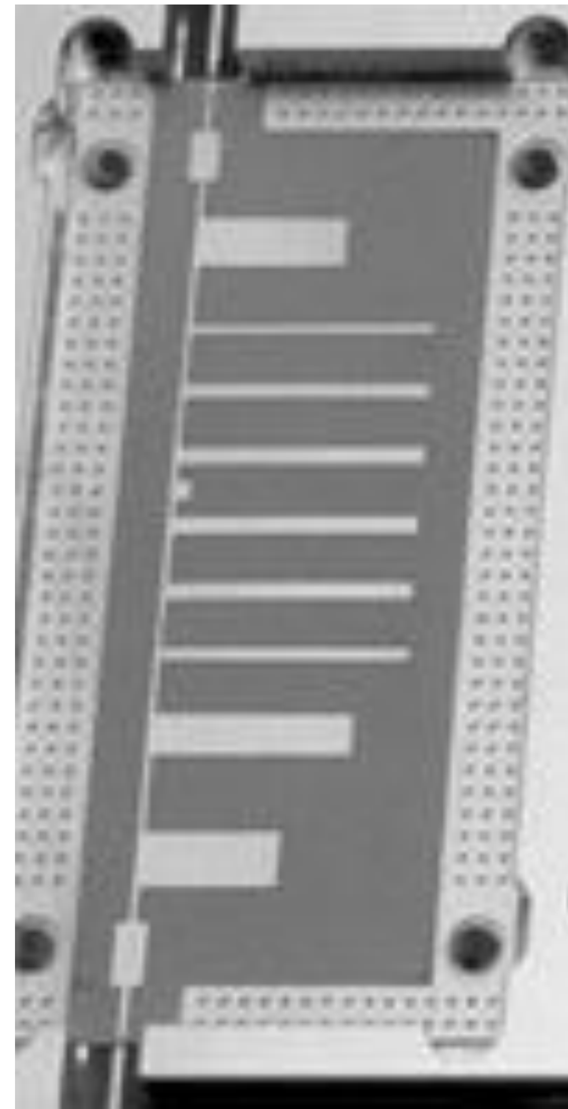


Figure 8.55
Courtesy of LNX Corporation, Salem, N.H.







Impedance and Admittance Inverters

- Richards' transformation and Kuroda's identities are useful especially for low-pass filters in technologies where the series stubs would be very difficult/ impossible to implement (microstrip)
- In the case of other filters (example 3rd order BPF):
  - series inductance can be implemented using K1-K2
  - series capacitance cannot be implemented using shunt stubs

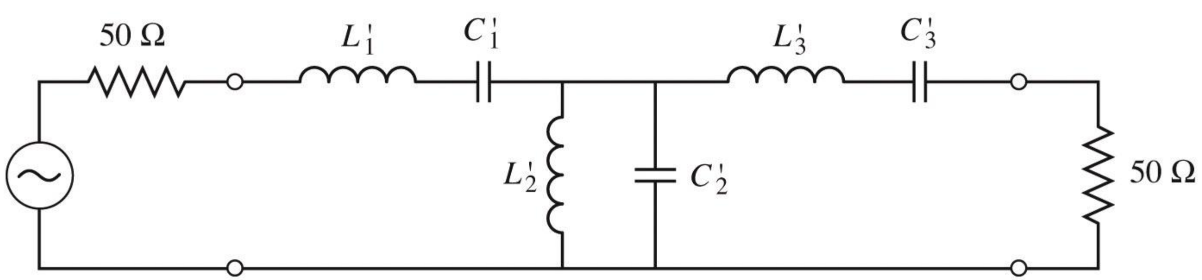


Figure 8.32

Impedance and Admittance Inverters

- For cases where Richards + Kuroda do not offer practical solutions we use circuits called **impedance and admittance inverters**

$$Z_{in} = \frac{K^2}{Z_L} \qquad\qquad Y_{in} = \frac{J^2}{Y_L}$$

Impedance inverters

Admittance inverters

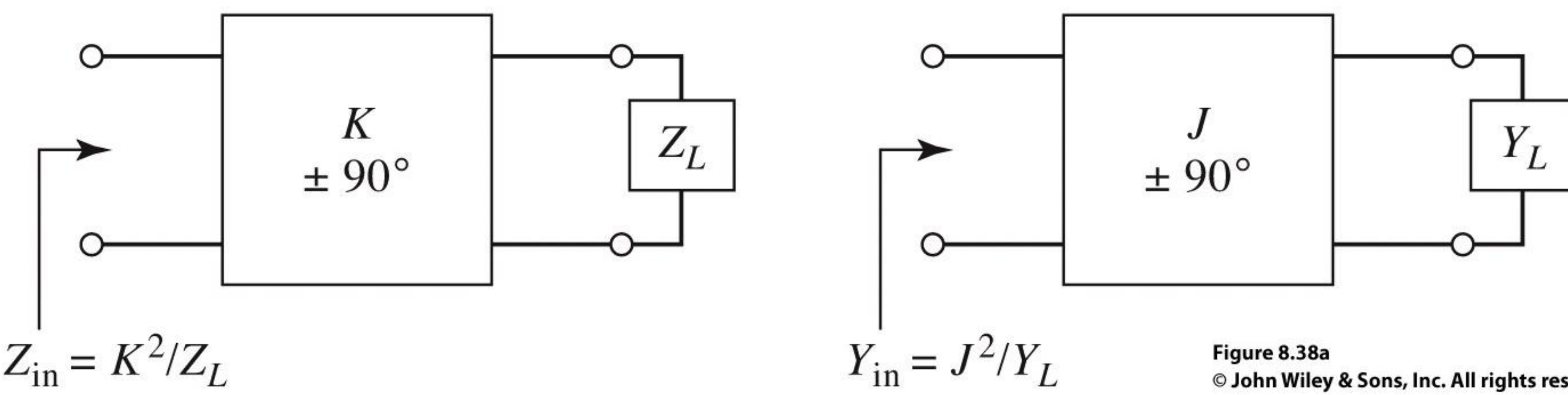
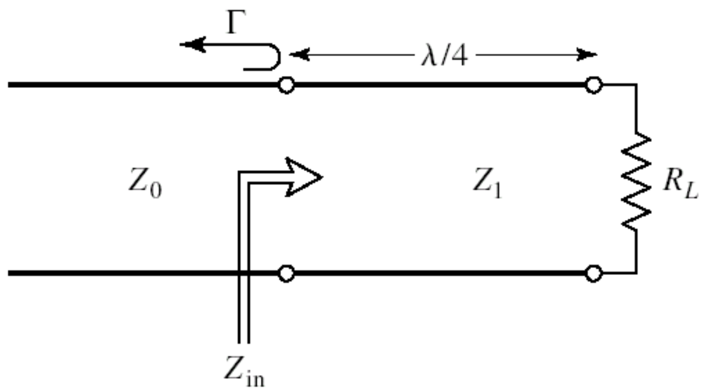


Figure 8.38a

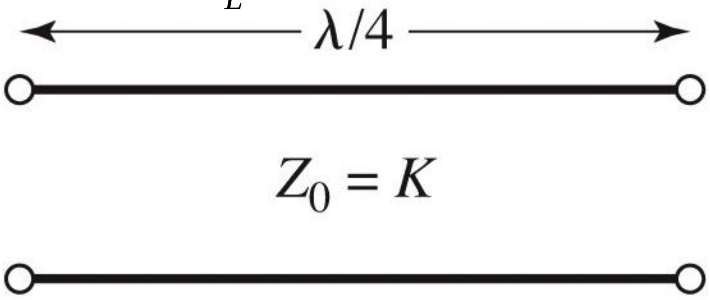
Impedance and Admittance Inverters

- The simplest example of impedance and admittance inverter is the **quarter-wave transformer** (L4)



$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$Z_{in} = \frac{K^2}{Z_L}$$



$$Y_{in} = \frac{J^2}{Y_L}$$

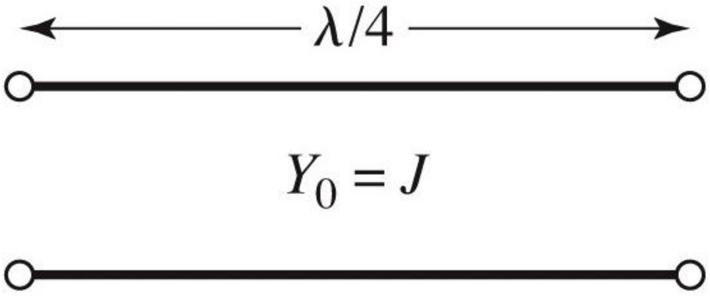
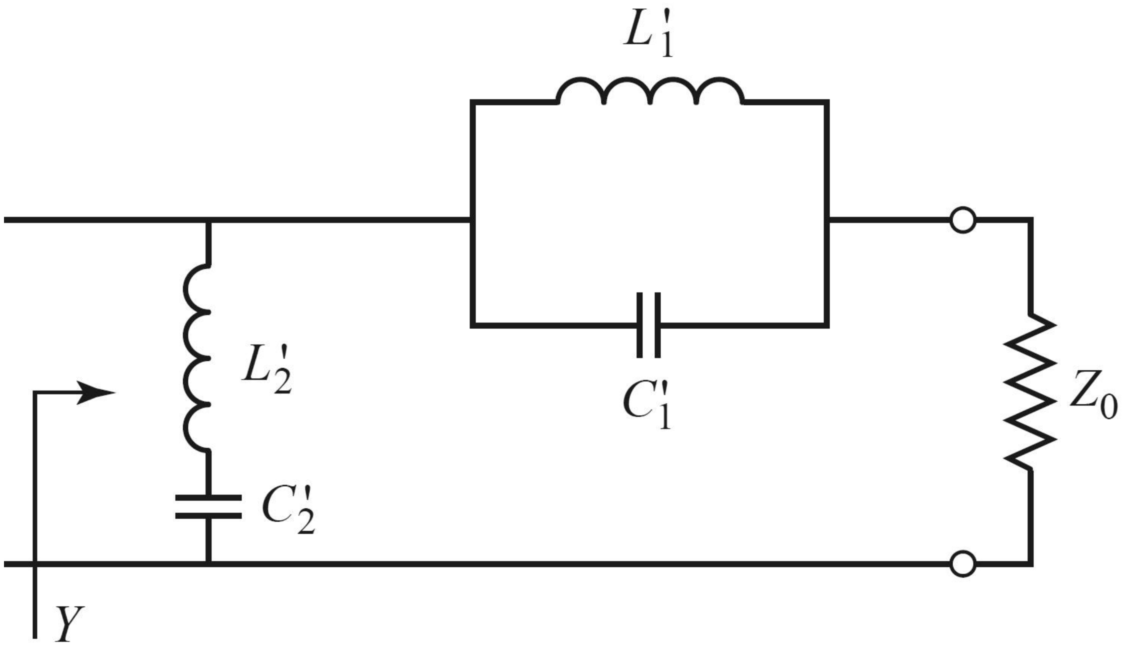


Figure 8.38b

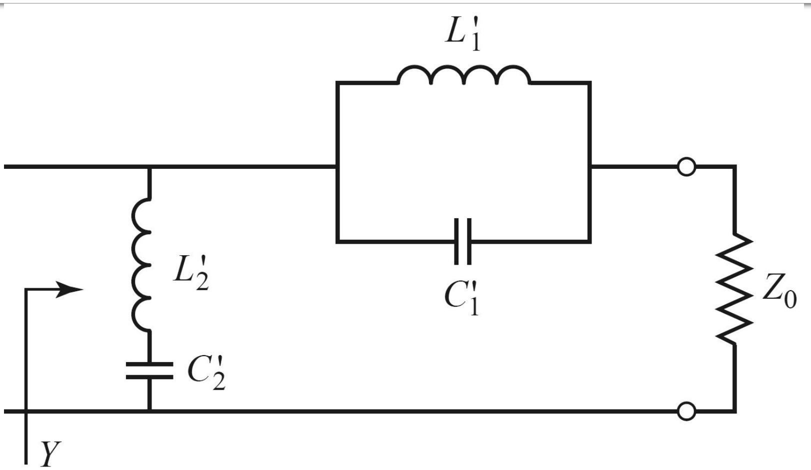
Impedance and Admittance Inverters

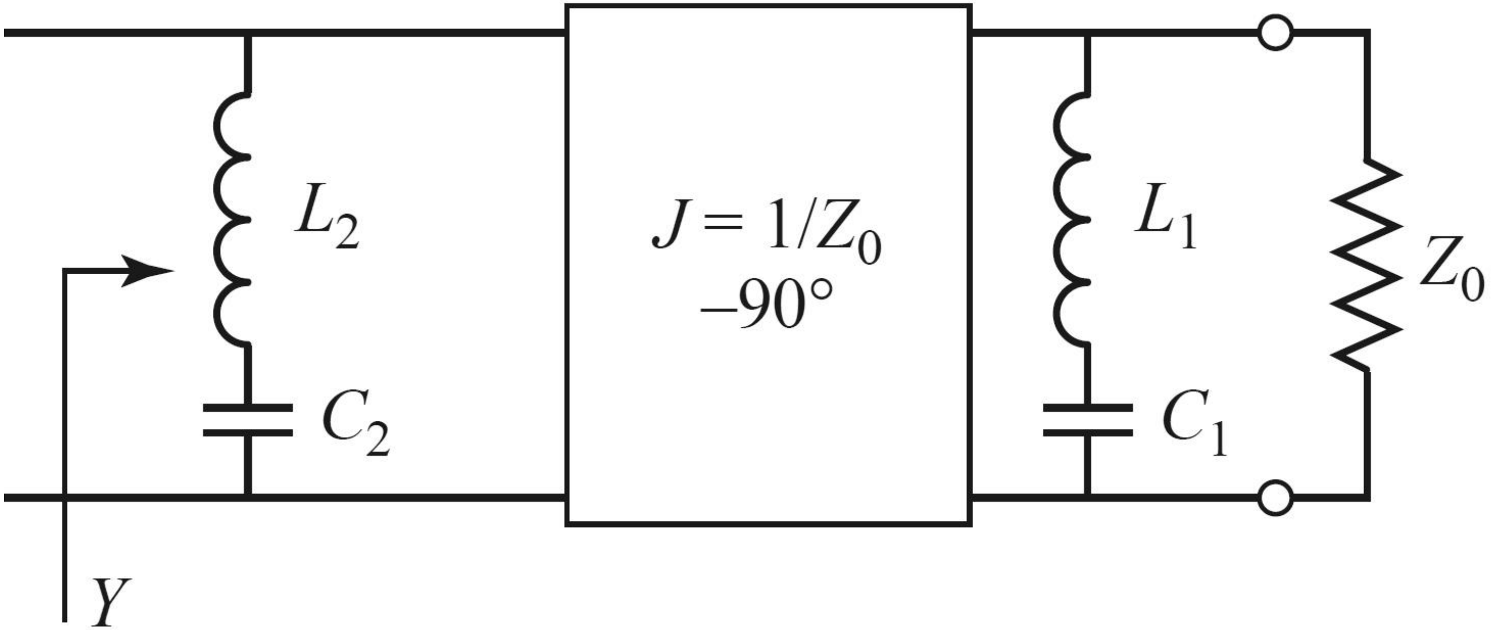
- Impedance/admittance inverters can be used to change the structure of a designed filter to a realizable form
- For example a 2nd order BSF



Impedance and Admittance Inverters

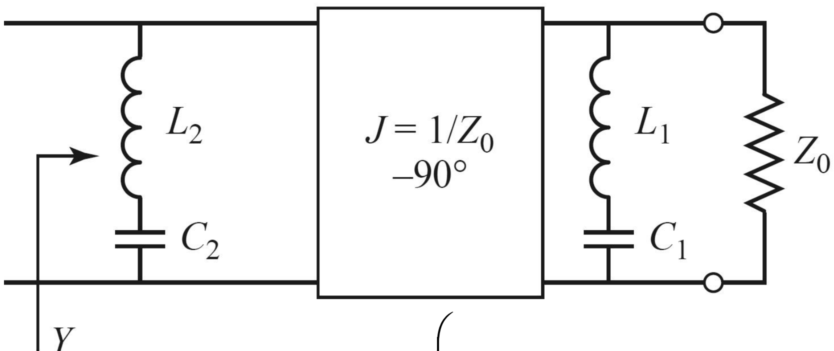
- The series elements can be eliminated/replaced using an admittance inverter

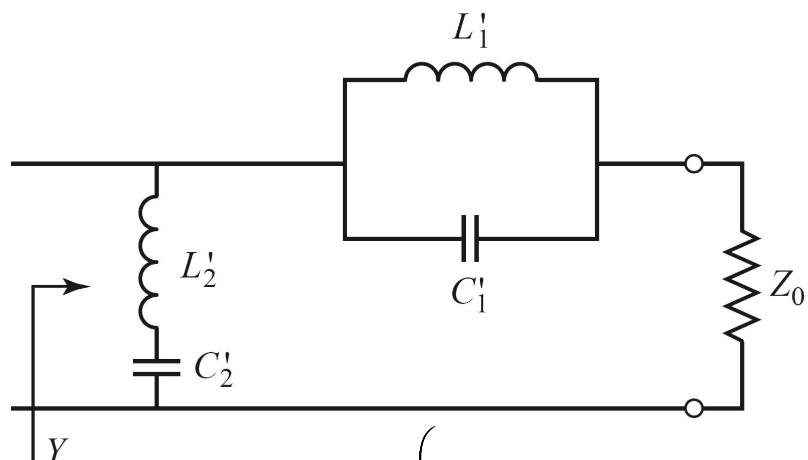




Impedance and Admittance Inverters

- The equivalence of the two schematics (when looking from the left) is proofed by obtaining the same input admittance





$$Y = \frac{1}{j\cdot\omega\cdot L_2 + \dfrac{1}{j\cdot\omega\cdot C_2}} + \frac{1}{Z_0^2}\cdot\left(\frac{1}{j\cdot\omega\cdot L_1 + \dfrac{1}{j\cdot\omega\cdot C_1}} + \frac{1}{Z_0}\right)^{-1}$$

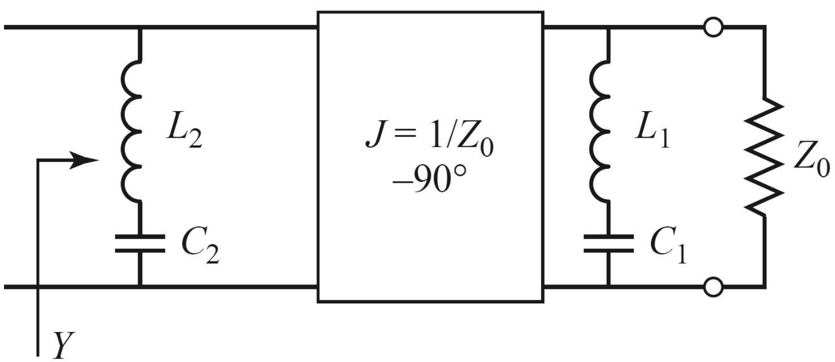
$$Y' = \frac{1}{j\cdot\omega\cdot L_2' + \dfrac{1}{j\cdot\omega\cdot C_2'}} + \left(\frac{1}{j\cdot\omega\cdot C_1' + \dfrac{1}{j\cdot\omega\cdot L_1'}} + Z_0\right)^{-1}$$

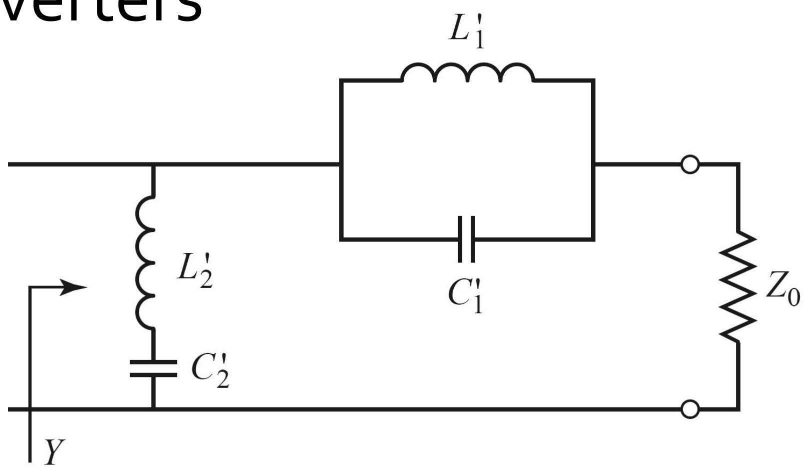
$$L_n\cdot C_n = L_n'\cdot C_n' = \frac{1}{\omega_0^2} \quad\Rightarrow\quad \begin{array}{c} \dfrac{1}{Z_0^2}\cdot\sqrt{\dfrac{L_1}{C_1}} = \sqrt{\dfrac{C_1'}{L_1'}} \\ \sqrt{\dfrac{L_2}{C_2}} = \sqrt{\dfrac{L_2'}{C_2'}} \end{array} \quad\Rightarrow\quad Y = Y'$$

- A similar result can be obtained for a bandpass filter

Impedance and Admittance Inverters

- The complete equivalence (when looking from both sides) is obtained by enclosing the series LC circuit between two admittance inverters

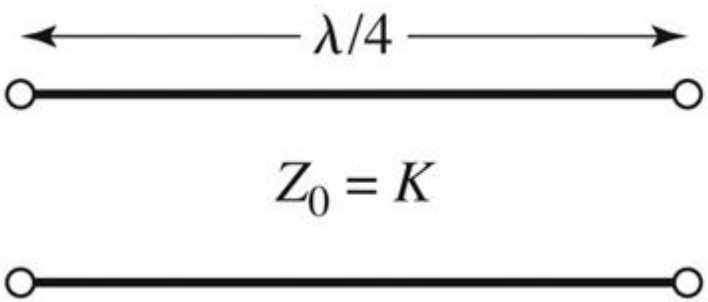


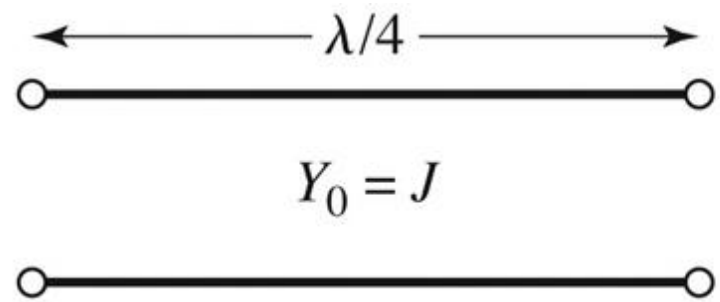


- A **series LC** circuit inserted in series in the circuit can be replaced by a **shunt LC** circuit inserted in parallel enclosed between 2 admittance inverters
- A **shunt LC** circuit inserted in series in the circuit can be replaced by a **series LC** circuit inserted in parallel enclosed between 2 admittance inverters

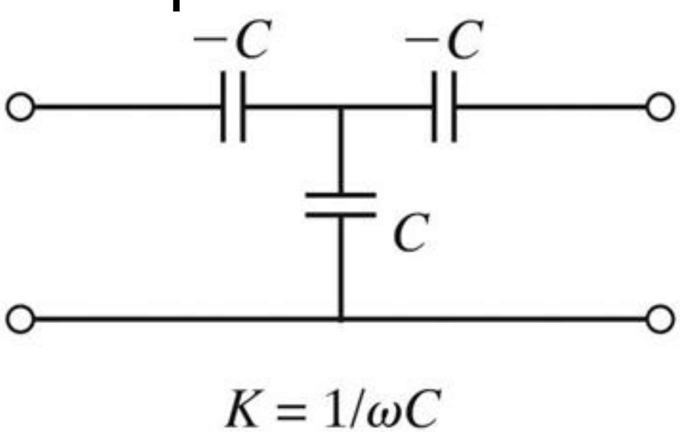
Practical implementations of impedance/admittance inverters

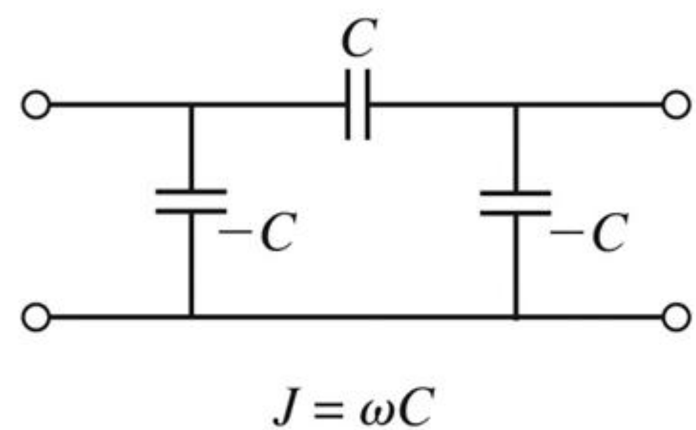
- Most often the quarter-wave transformer is used





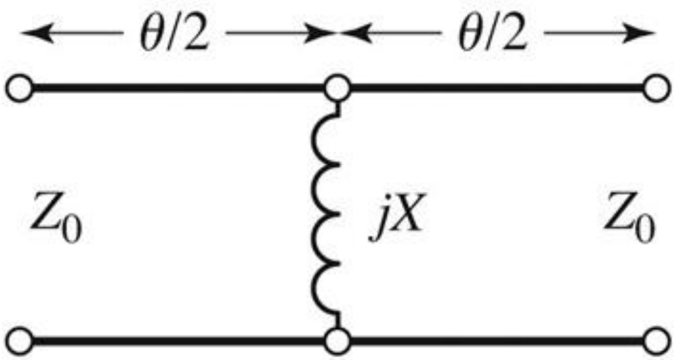
- Implementation with capacitor networks

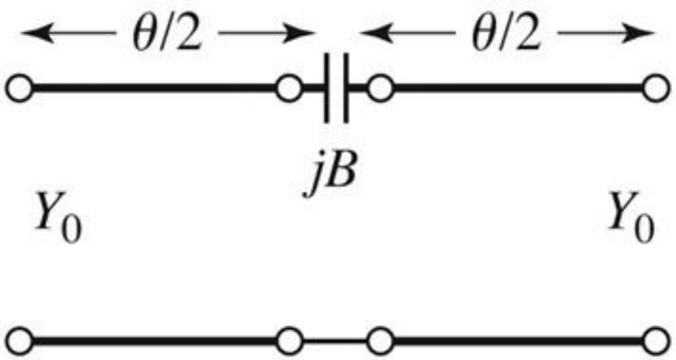




Practical implementations of impedance/admittance inverters

- Implementation with transmission lines and reactive elements





$$K = Z_0 \cdot \tan\left|\frac{\theta}{2}\right|$$

$$X = \frac{K}{1-\left(\frac{K}{Z_0}\right)^2}$$

$$\theta = -\tan^{-1}\frac{2\cdot X}{Z_0}$$

$$J = Y_0 \cdot \tan\left|\frac{\theta}{2}\right|$$

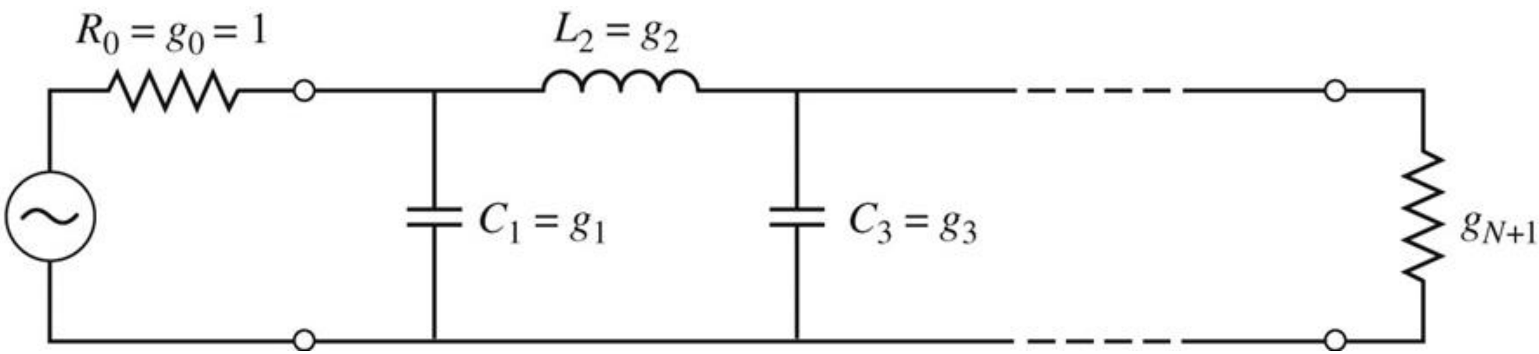
$$B = \frac{J}{1-\left(\frac{J}{Y_0}\right)^2}$$

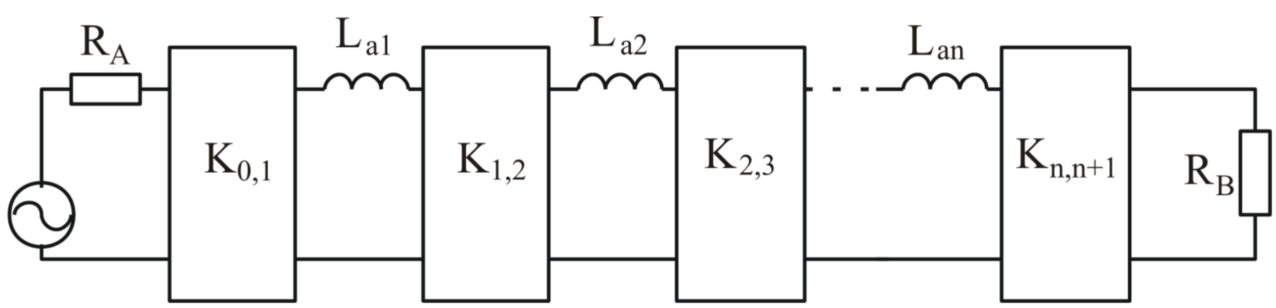
$$\theta = -\tan^{-1}\frac{2\cdot B}{Y_0}$$

$$\theta < 0$$

Prototype filters using inverters

- Using impedance/admittance inverters we can implement prototype filters using a single type of reactive elements
  - Shunt C replaced by series L enclosed between 2 inverters

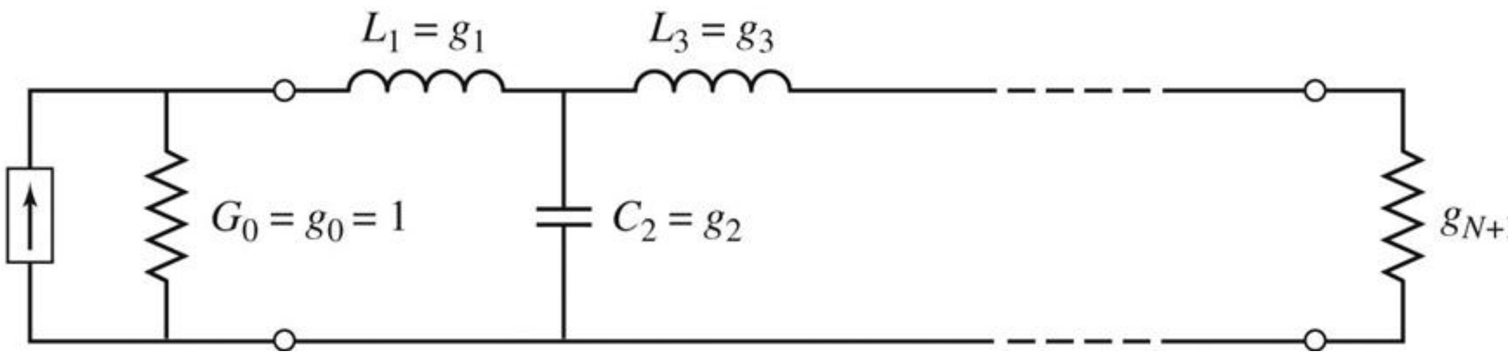


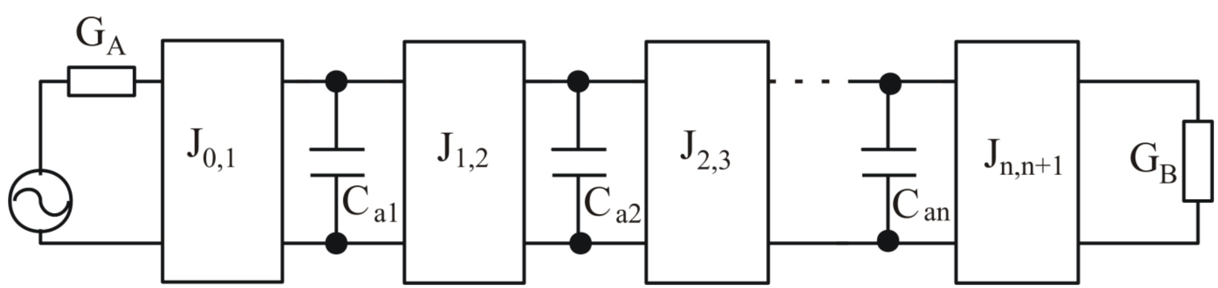


$$K_{0,1} = \sqrt{\frac{R_A \cdot L_{a,1}}{g_0 \cdot g_1}} \qquad K_{k,k+1}\Big|_{k=\overline{1,n-1}} = \sqrt{\frac{L_{a,k} \cdot L_{a,k+1}}{g_k \cdot g_{k+1}}} \qquad K_{n,n+1} = \sqrt{\frac{L_{a,n} \cdot R_B}{g_n \cdot g_{n+1}}}$$

Prototype filters using inverters

- Using impedance/admittance inverters we can implement prototype filters using a single type of reactive elements
  - Series L replaced by shunt C enclosed between 2 inverters





$$J_{0,1} = \sqrt{\frac{G_A \cdot C_{a,1}}{g_0 \cdot g_1}} \qquad J_{k,k+1}\Big|_{k=\overline{1,n-1}} = \sqrt{\frac{C_{a,k} \cdot C_{a,k+1}}{g_k \cdot g_{k+1}}} \qquad J_{n,n+1} = \sqrt{\frac{C_{a,n} \cdot g_B}{g_n \cdot g_{n+1}}}$$

Prototype filters using inverters

- For prototype filters using inverters formulas we have $2 \cdot N+1$ parameters and $N+1$ equations (to ensure the equivalence of the 2 schematics) so N parameters can be chosen freely
  - convenient values for the reactance can be chosen, and the required inverters will be computed from the equivalence equations or,
  - convenient inverters can be chosen, and the required reactance values will be computed from the equivalence equations

BPF and BSF using inverters

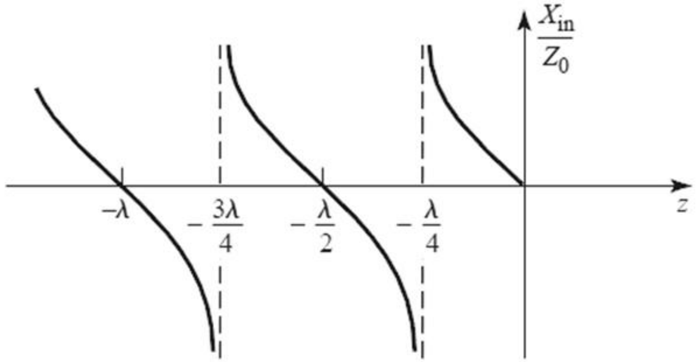
- The same principle can be applied to the BPF and BSF filters, those can be implemented using N+1 inverters and N resonators (**series or shunt LC circuits** with resonant frequency ω_o) connected either in series or in parallel enclosed between 2 inverters
  - BPF are implemented with
    - series LC circuits connected in series between inverters
    - shunt LC circuits connected in parallel between inverters
  - BSF are implemented with
    - shunt LC circuits connected in series between inverters
    - series LC circuits connected in parallel between inverters

Lines as resonators

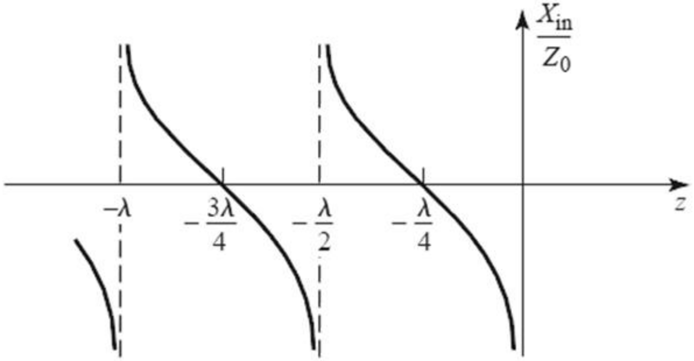
- The impedance of short-circuited or open-circuited line (stub) shows a resonant behavior that can be used to implement required resonators

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

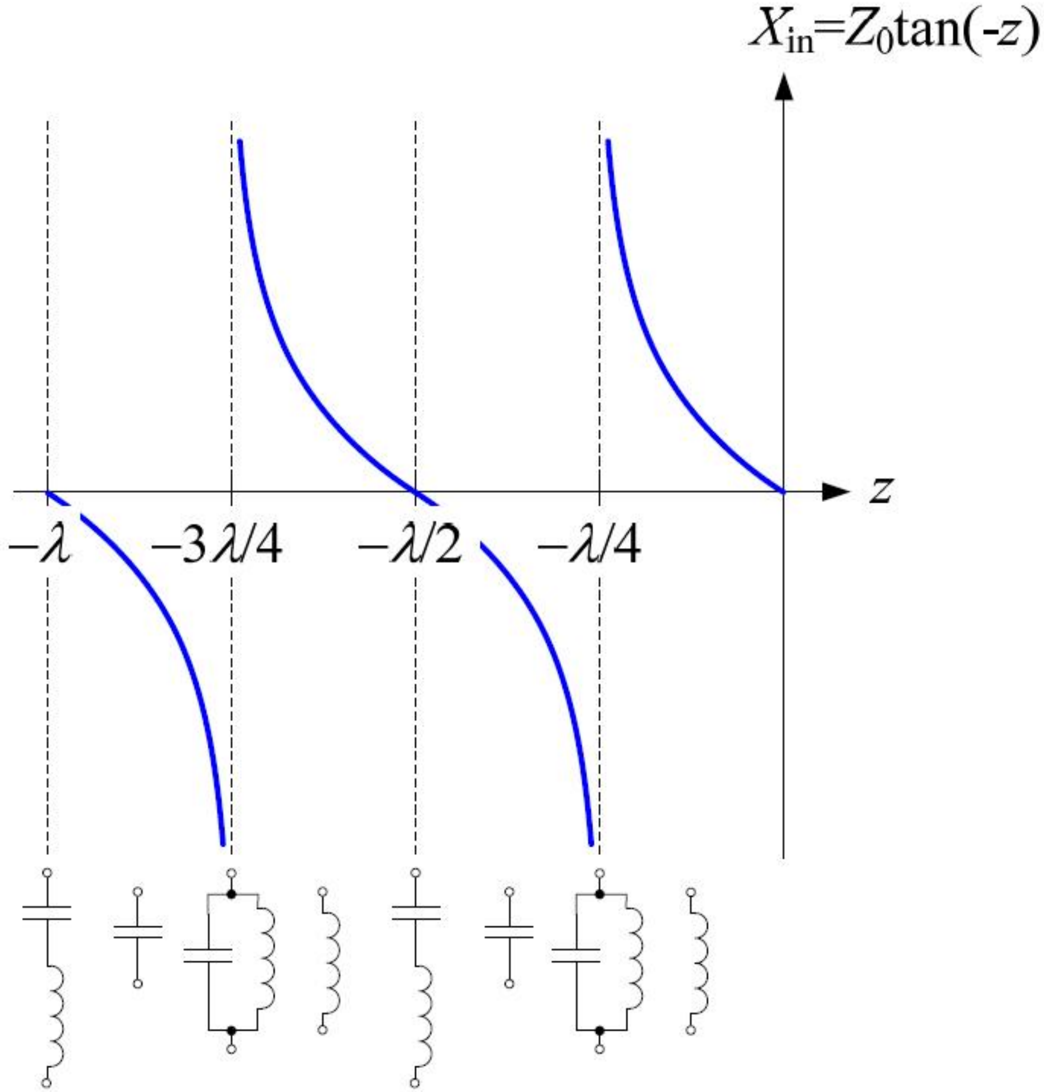


$$Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$



Lines as resonators

- Short-circuited line
- For the frequency at which l = λ/4 (ω_o) the line behaves as an shunt LC resonator circuit
  - the line shows capacitive behavior for lower frequencies (l>λ/4)
  - the line shows inductive behavior for higher frequencies (l<λ/4)
- Similar discussion for the open circuited line (equivalent to a series LC resonator around the frequency at which l=λ/4)



BPF/BSP design formulas

- When the admittance inverters are implemented with quarter-wave transformers with Zo characteristic impedance
  - BPF – short-circuited shunt stubs with l = λ/4

$$Z_{0n} \approx \frac{\pi \cdot Z_0 \cdot \Delta}{4 \cdot g_n}$$

  - BSF – open-circuited shunt stubs with l = λ/4

$$Z_{0n} \approx \frac{4 \cdot Z_0}{\pi \cdot g_n \cdot \Delta}$$

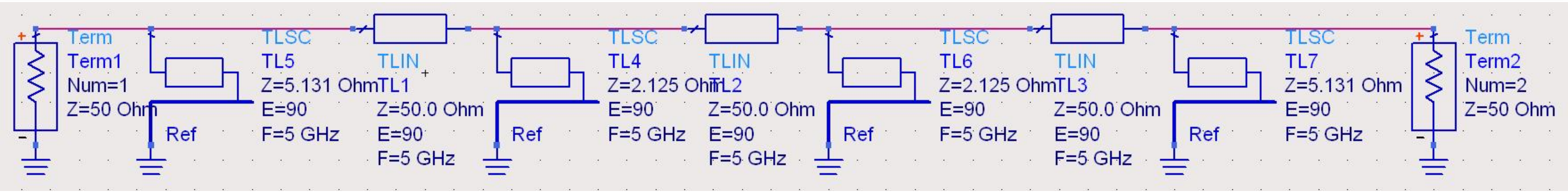
Example

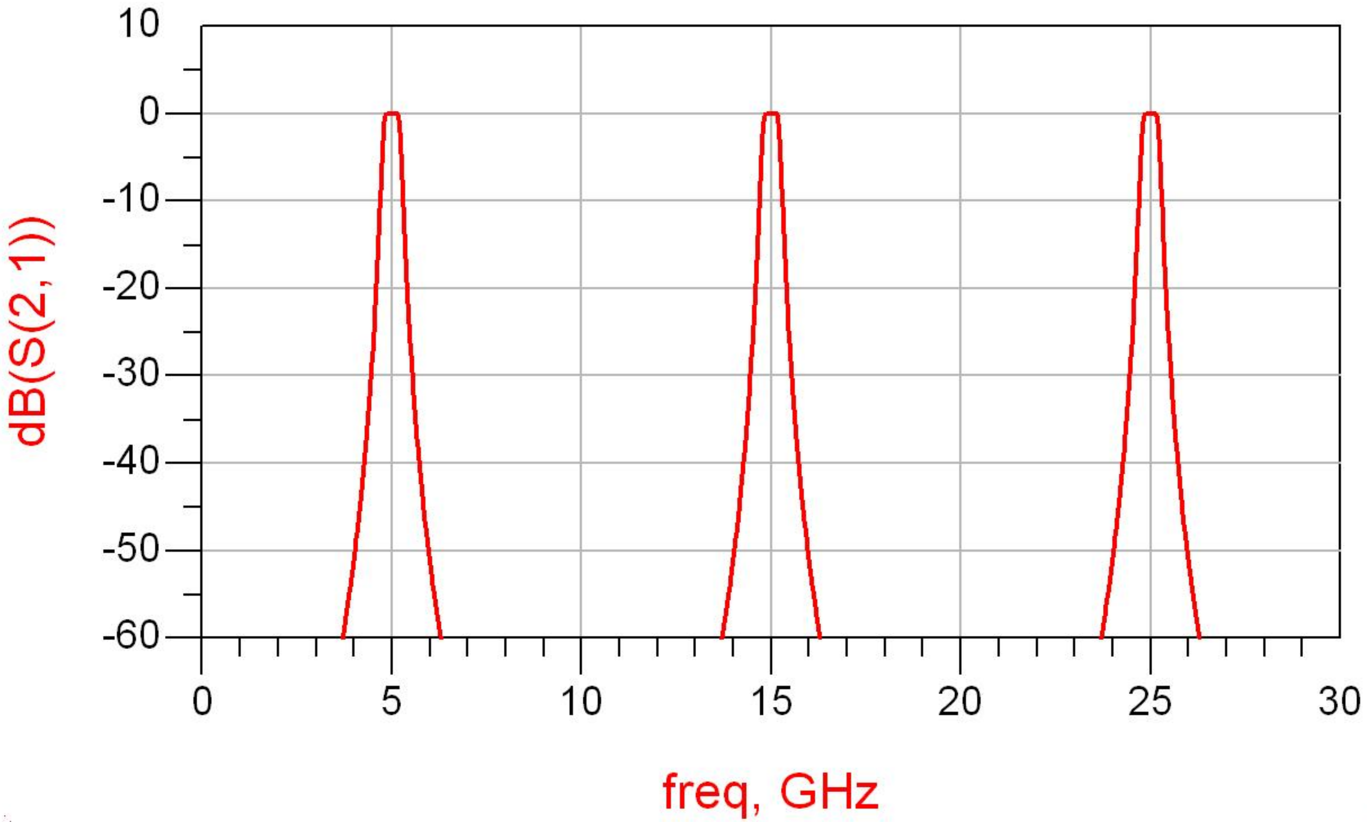
- Similar to a project assignment
- Follows the amplifier designed as in L8
- 4^{th} order bandpass filter, fo = 5GHz, fractional bandwidth of the passband 10 %
- maximally flat table or formulas for g_n:

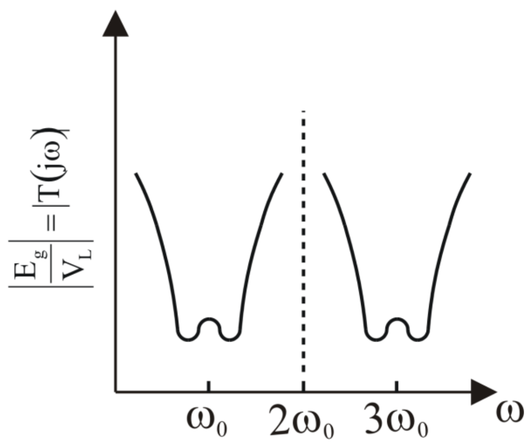
n	g_n	$Z_{on}(\Omega)$
1	0.7654	5.131
2	1.8478	2.125
3	1.8478	2.125
4	0.7654	5.131

$$Z_{0n} \approx \frac{\pi \cdot Z_0 \cdot \Delta}{4 \cdot g_n}$$

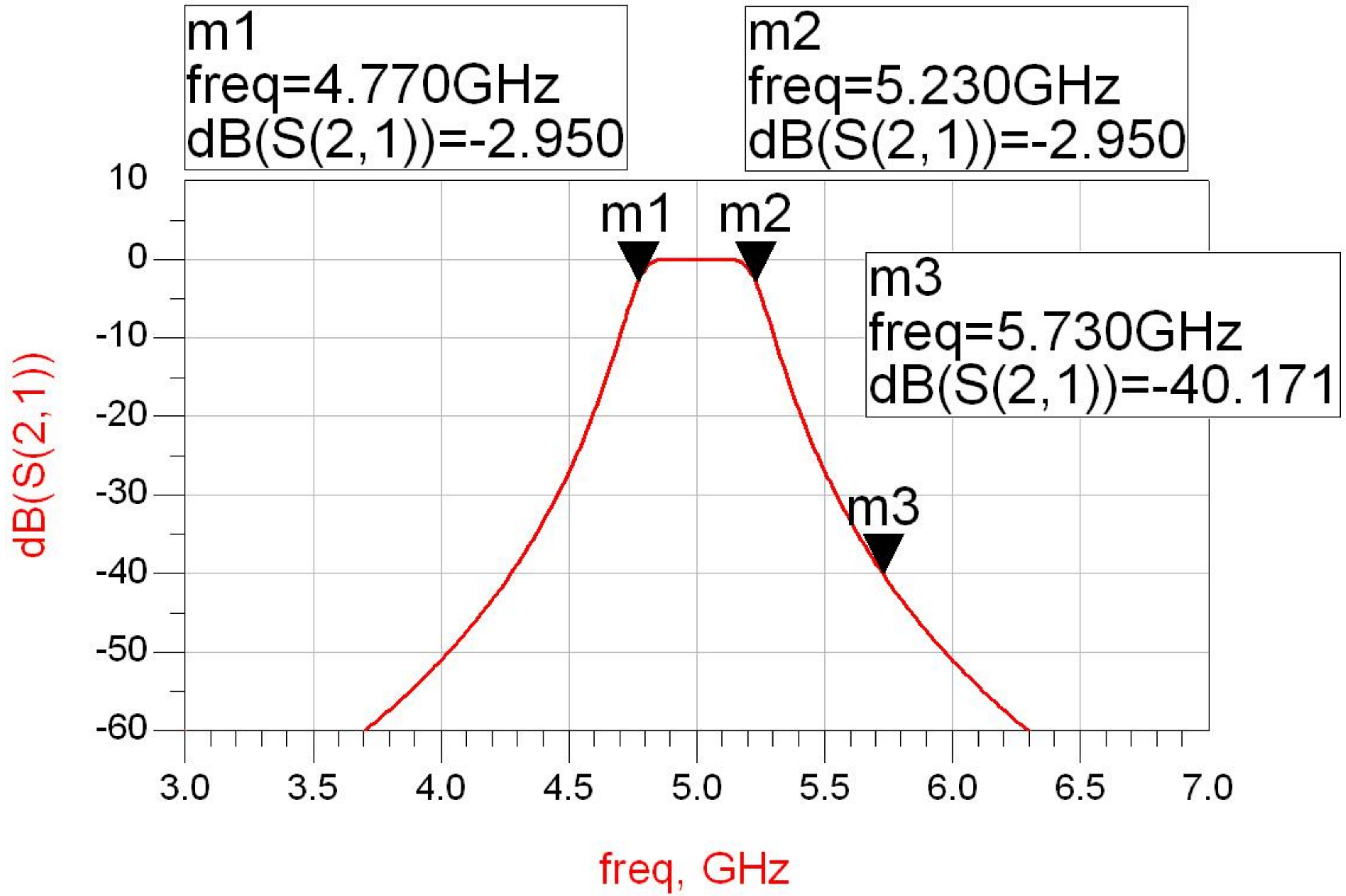
ADS – BPF


Term
Term1
Num=1
Z=50 Ohm
TLSC
TL5
Z=5.131 Ohm
E=90
F=5 GHz
Ref
TLIN
TL1
Z=50.0 Ohm
E=90
F=5 GHz
TLSC
TL4
Z=2.125 Ohm
E=90
F=5 GHz
Ref
TLIN
TL2
Z=50.0 Ohm
E=90
F=5 GHz
TLSC
TL6
Z=2.125 Ohm
E=90
F=5 GHz
Ref
TLIN
TL3
Z=50.0 Ohm
E=90
F=5 GHz
TLSC
TL7
Z=5.131 Ohm
E=90
F=5 GHz
Ref
Term
Term2
Num=2
Z=50 Ohm


dB(S(2,1))
10
0
-10
-20
-30
-40
-50
-60
0
5
10
15
20
25
30
freq, GHz


$\left|\frac{E_g}{V_L}\right| = |T(j\omega)|$
ω_0
$2\omega_0$
$3\omega_0$
ω

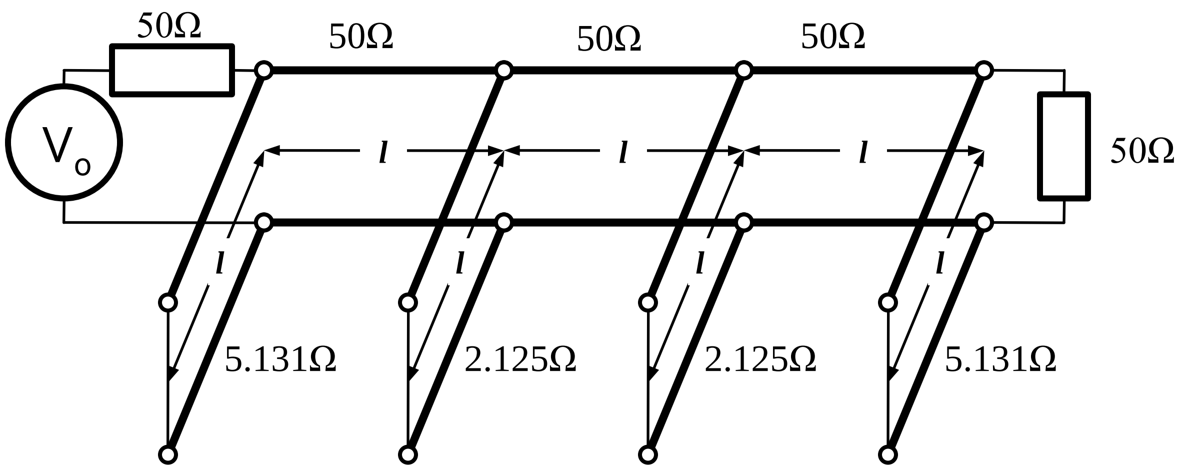
ADS – BPF



m1
freq=4.770GHz
dB(S(2,1))=-2.950
m2
freq=5.230GHz
dB(S(2,1))=-2.950
m3
freq=5.730GHz
dB(S(2,1))=-40.171
10
0
-10
-20
-30
-40
-50
-60
dB(S(2,1))
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
freq, GHz

Example

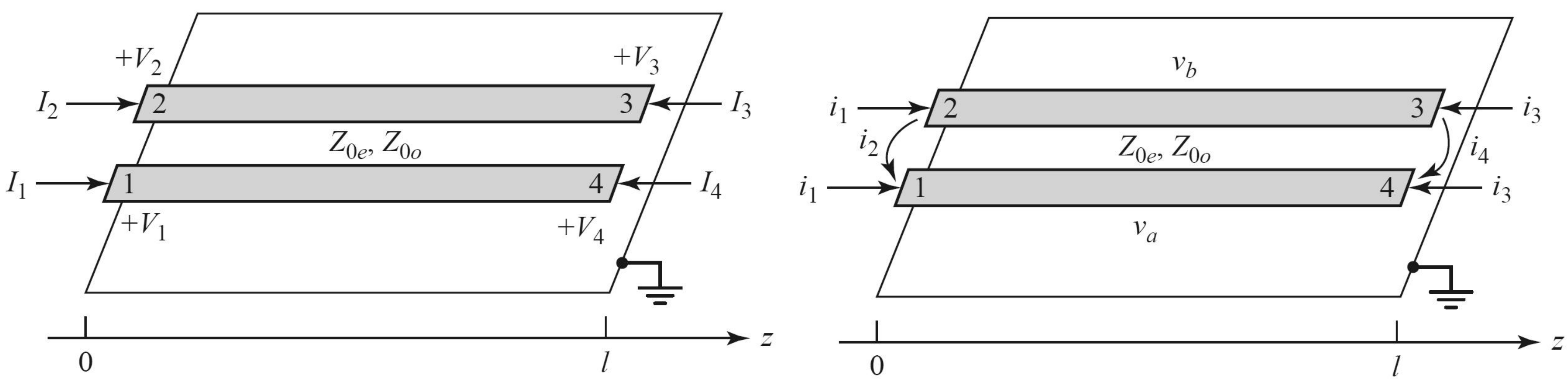
$$l = \frac{\lambda}{4} \quad \Rightarrow \quad \beta \cdot l = \frac{\pi}{2}$$



- Disadvantages of the filters using impedance inverters and lines as resonators:
  - short-circuited stubs (via-hole) for BPF
  - often the **characteristic impedances** for the stubs have **values** difficult to implement (2.125Ω)

Coupled Line Filters

- A parallel coupled line section model is obtained by even/odd mode analysis
- Even and odd modes are characterized by the characteristic even/odd mode impedances whose required values will impose the lines' geometry (width / distance between lines, depending on the line technology we use)



Coupled Lines

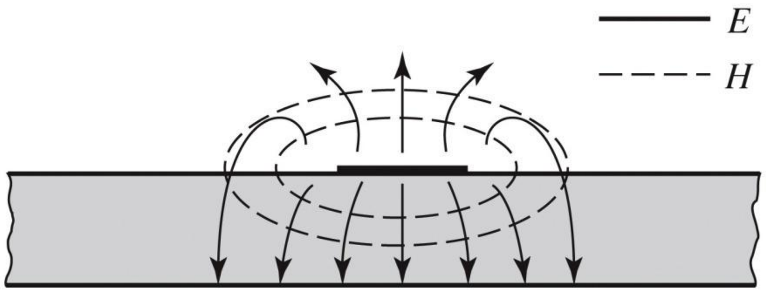
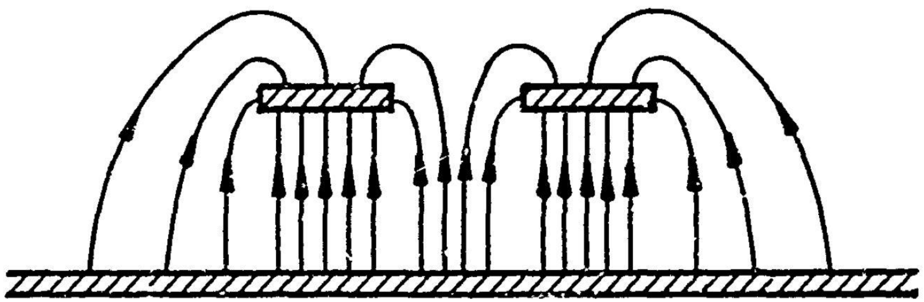
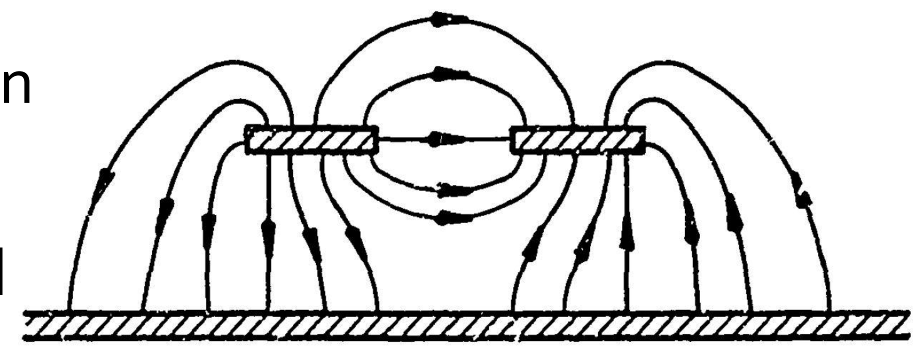


Figure 3.25b



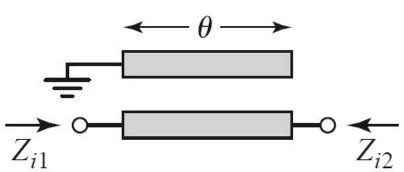
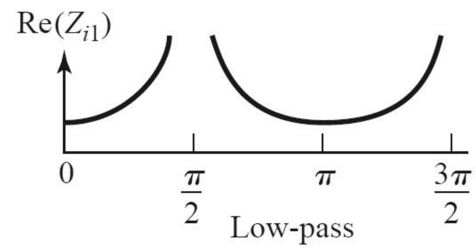
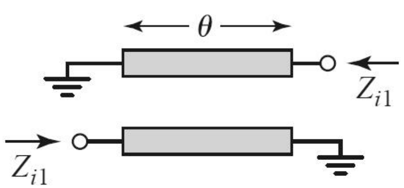
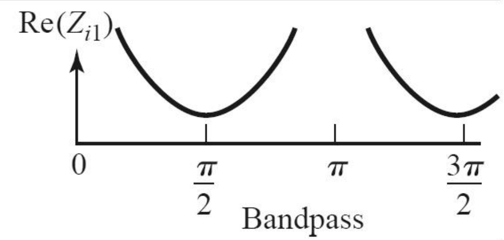
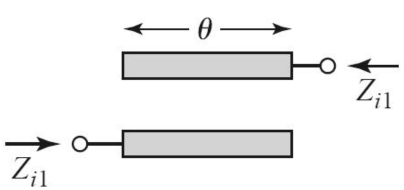
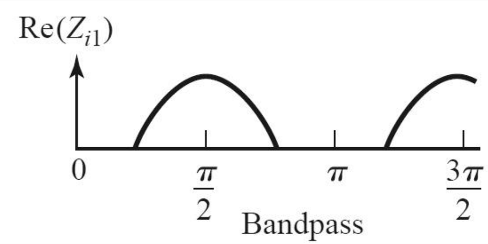
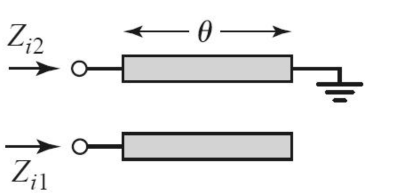
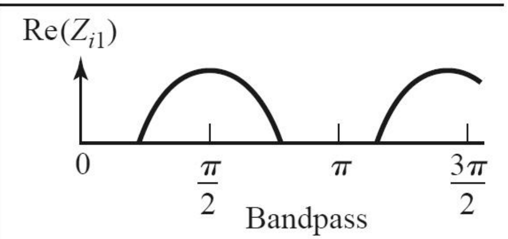
b) EVEN MODE ELECTRIC FIELD PATTERN (SCHEMATIC)



c) ODD MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

- Even mode - characterizes the common mode signal on the two lines
- Odd mode - characterizes the differential mode signal between the two lines
- Each of the two modes is characterized by **different** characteristic impedances

Coupled Line Filters

Circuit	Image Impedance	Response
	$Z_{i1} = \dfrac{2Z_{0e}Z_{0o}\cos\theta}{\sqrt{(Z_{0e}+Z_{0o})^2\cos^2\theta-(Z_{0e}-Z_{0o})^2}}$ $Z_{i2} = \dfrac{Z_{0e}Z_{0o}}{Z_{i1}}$	
	$Z_{i1} = \dfrac{2Z_{0e}Z_{0o}\sin\theta}{\sqrt{(Z_{0e}-Z_{0o})^2-(Z_{0e}+Z_{0o})^2\cos^2\theta}}$	
	$Z_{i1} = \dfrac{\sqrt{(Z_{0e}-Z_{0o})^2-(Z_{0e}+Z_{0o})^2\cos^2\theta}}{2\sin\theta}$	
	$Z_{i1} = \dfrac{\sqrt{Z_{0e}Z_{0o}}\sqrt{(Z_{0e}-Z_{0o})^2-(Z_{0e}+Z_{0o})^2\cos^2\theta}}{(Z_{0e}+Z_{0o})\sin\theta}$ $Z_{i2} = \dfrac{Z_{0e}Z_{0o}}{Z_{i1}}$	

Coupled Line Filters

- Bandpass filter with resonance at θ=π/2 (*l*=λ/4)

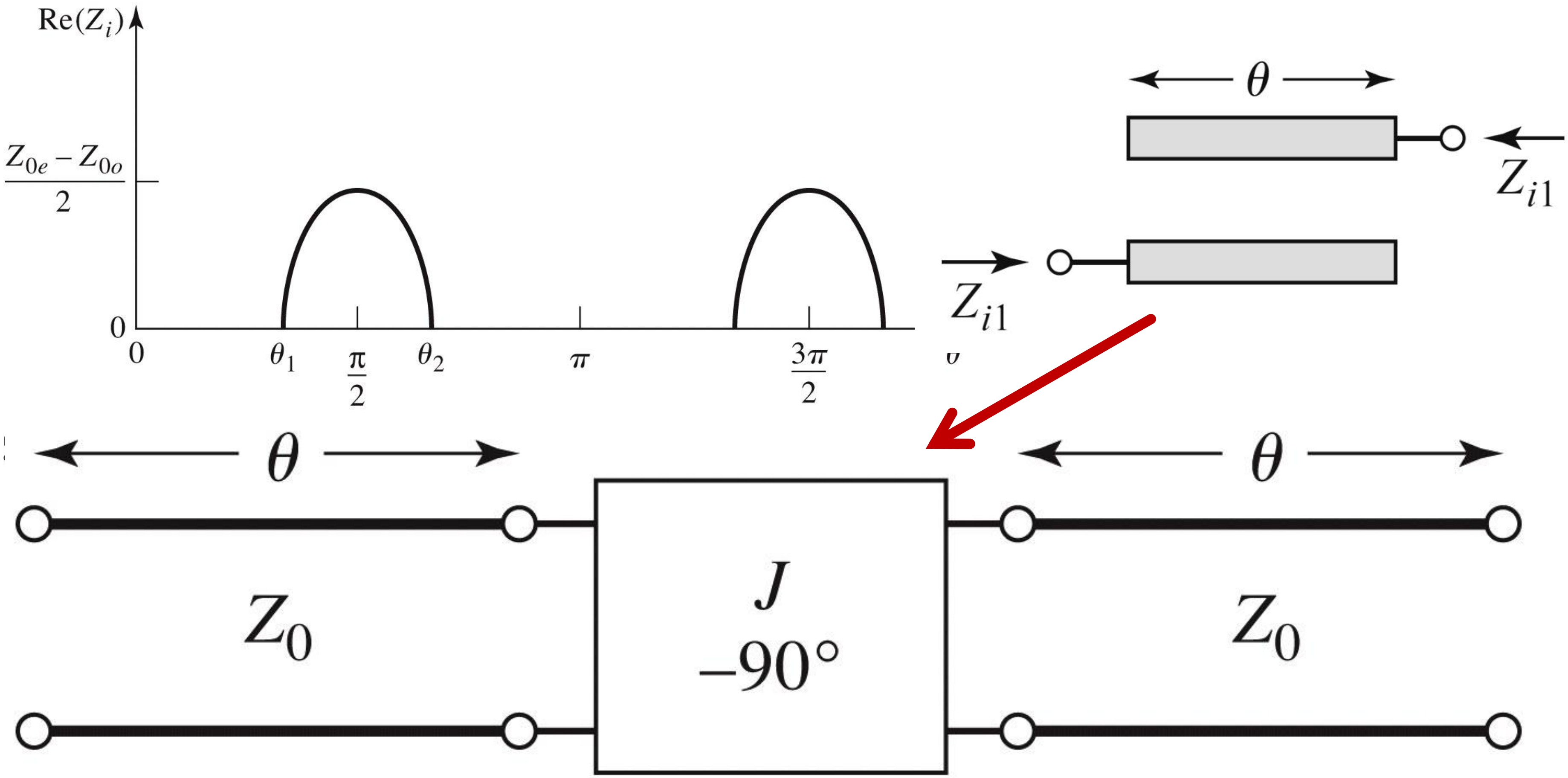
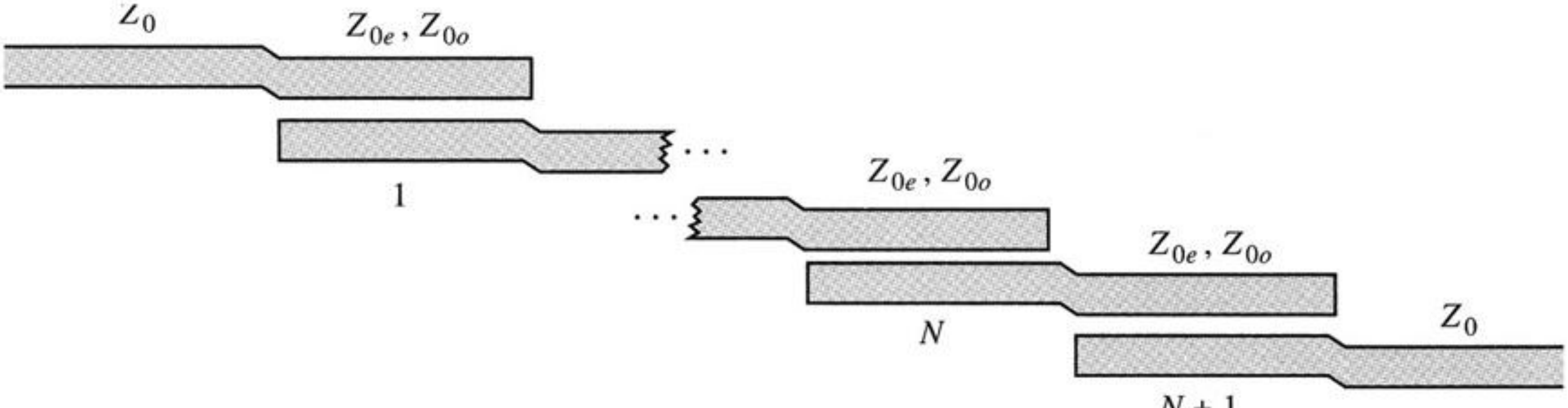


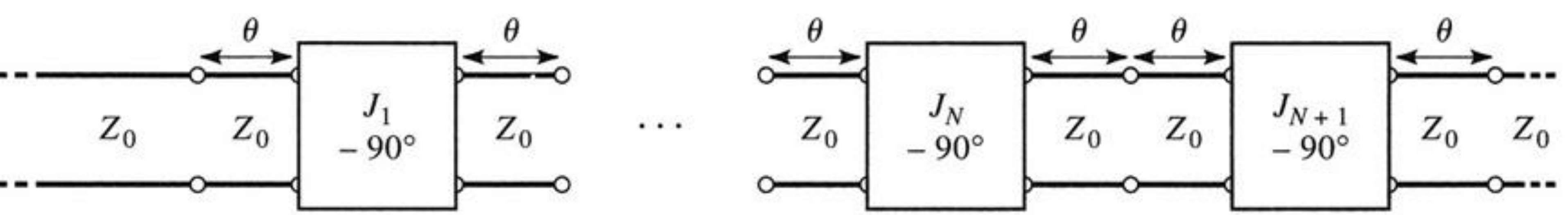
Figure 8.44

Coupled Line Filters

- We get a N^{th} order filter with N+1 parallel coupled line section

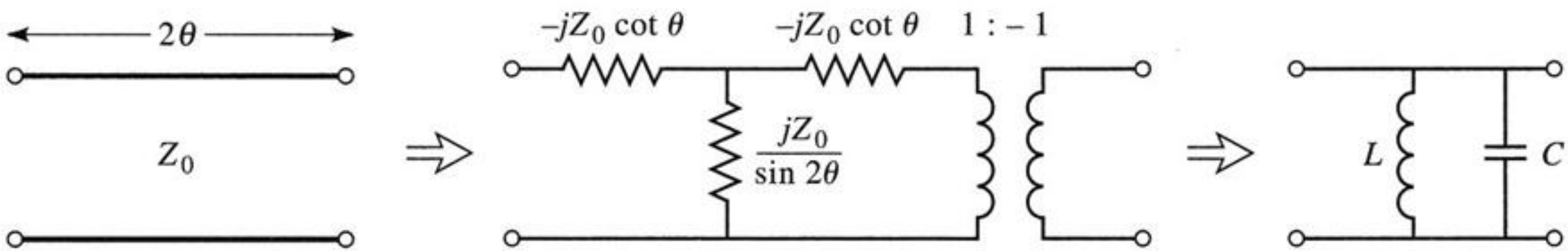


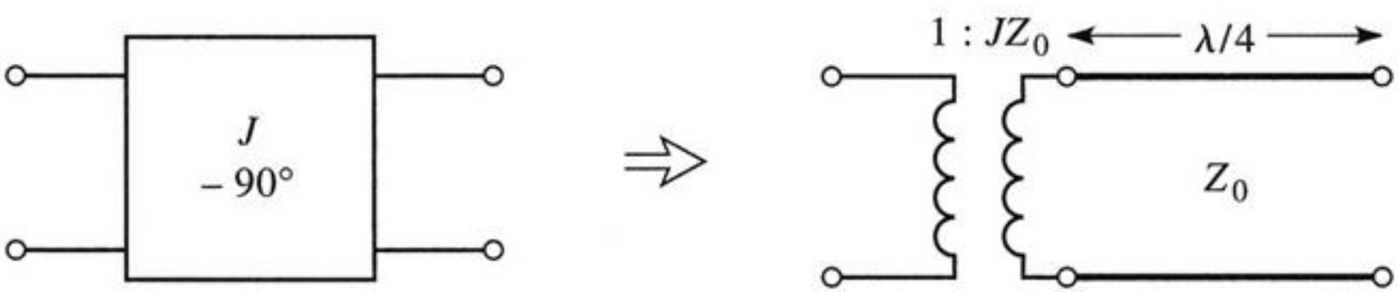
(a)



Coupled Line Filters

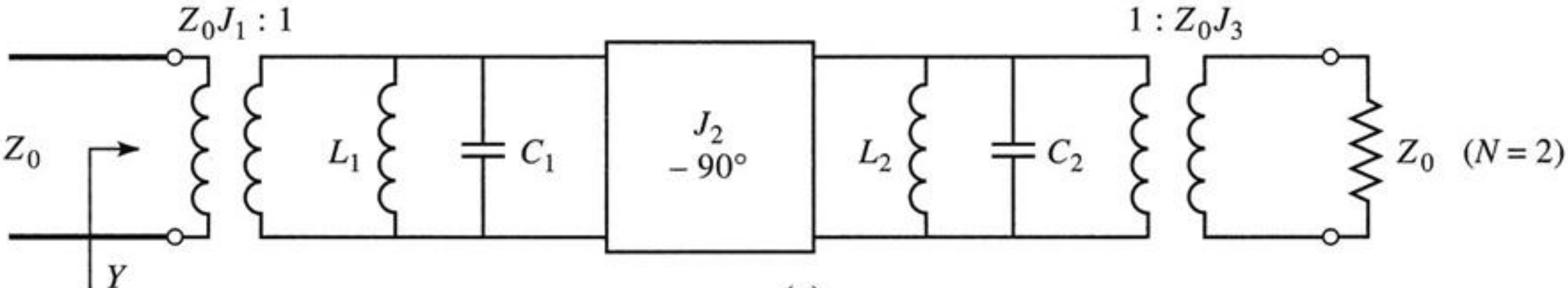
- Equivalent circuits for
  - transmission lines of length 2θ
  - admittance inverters



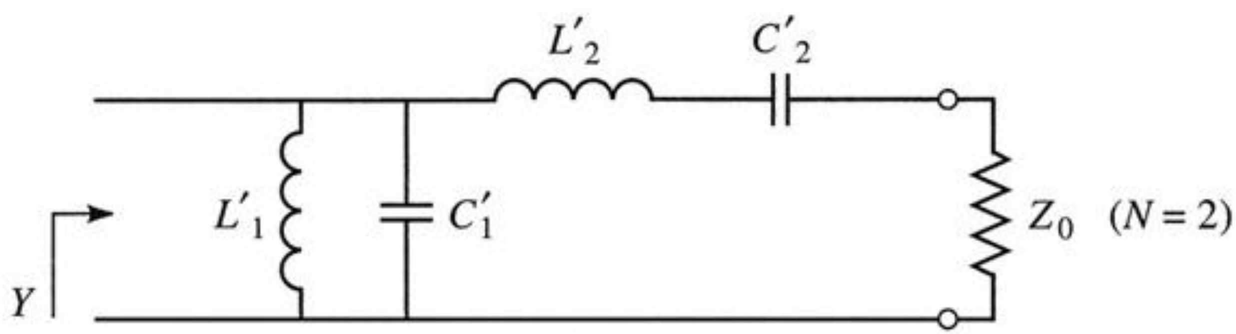


Coupled Line Filters N=2

- We get a 2nd order BPF behavior cu 3 coupled lines sections



(e)



(f)

Figure 8.45def

Coupled Line Filters design formulas

- Compute the inverters from prototype parameters

$$Z_0 \cdot J_1 = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_1}} \qquad Z_0 \cdot J_n = \frac{\pi \cdot \Delta}{2 \cdot \sqrt{g_{n-1} \cdot g_n}}, n = \overline{2, N} \qquad Z_0 \cdot J_{N+1} = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_N \cdot g_{N+1}}}$$

- Compute coupled line parameters Zoe/Zoo (all of length *l*=λ/4)

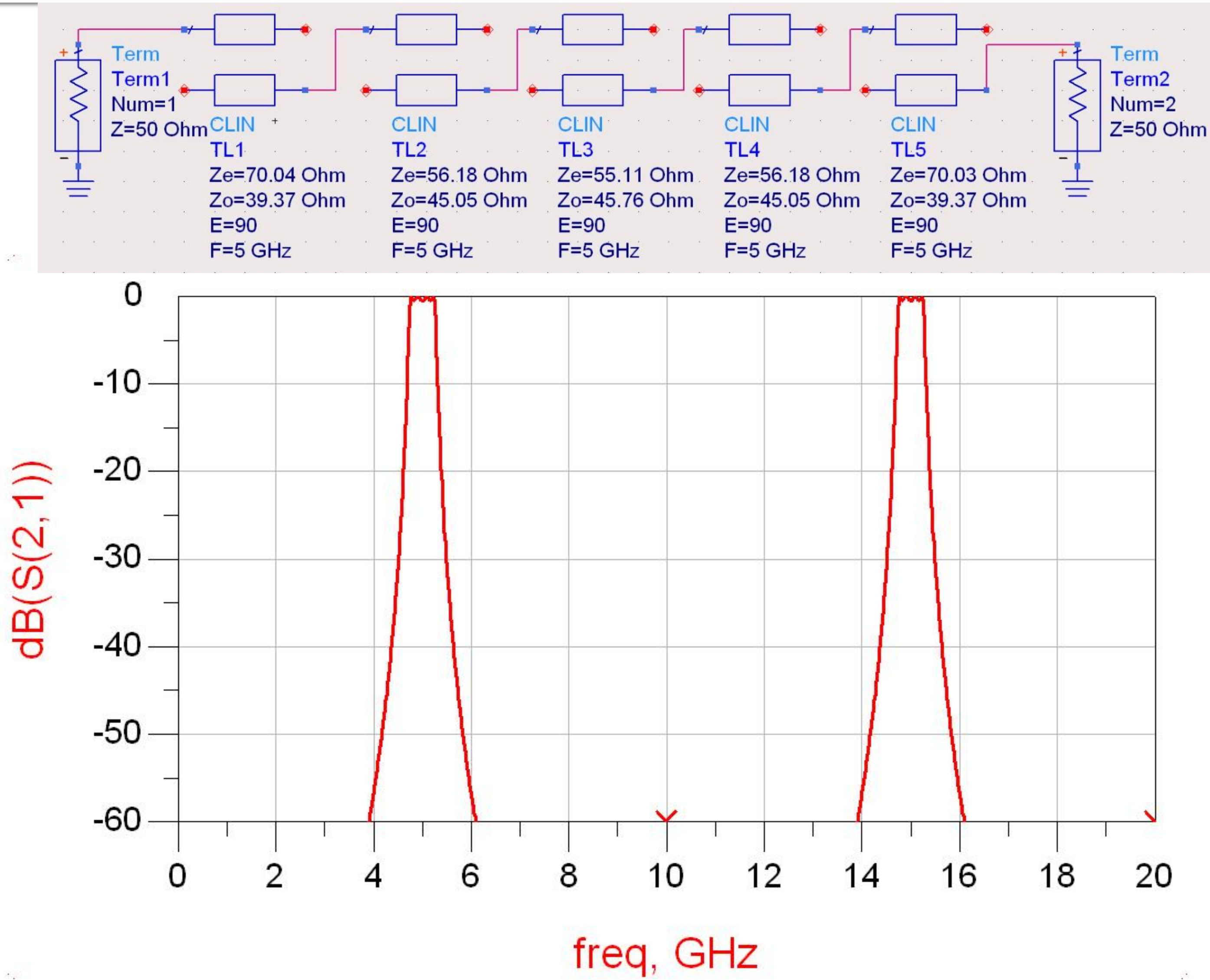
$$Z_{0e,n} = Z_0 \cdot \left[1 + J_n \cdot Z_0 + \left(J_n \cdot Z_0\right)^2\right]$$
$$Z_{0o,n} = Z_0 \cdot \left[1 - J_n \cdot Z_0 + \left(J_n \cdot Z_0\right)^2\right]$$
$$n = \overline{1, N+1}$$

Example

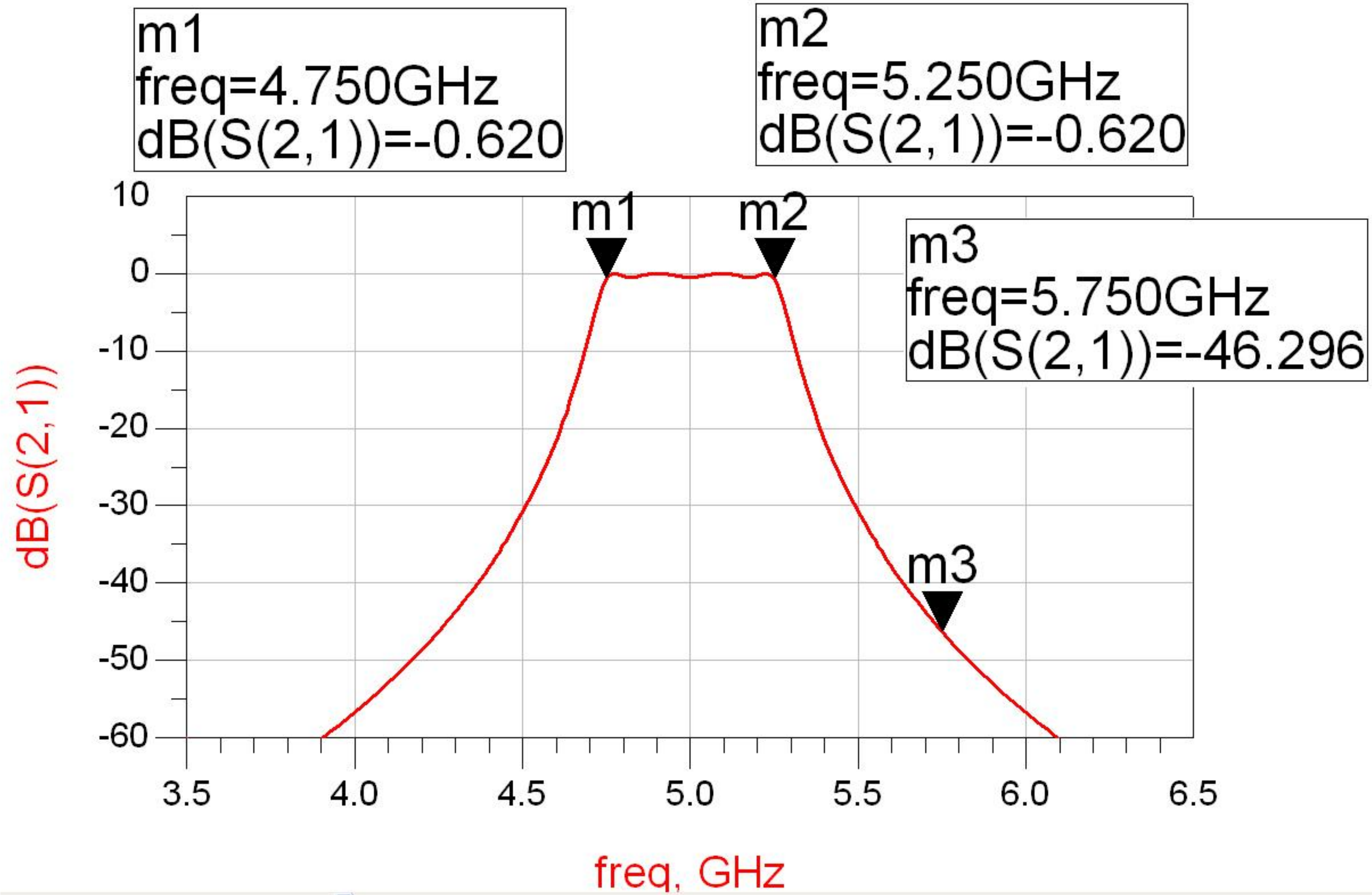
- Similar to a project assignment
- Follows the amplifier designed as in L8
- 4^{th} order bandpass filter, fo = 5GHz, fractional bandwidth of the passband 10 %
- 0.5dB equal-ripple table for g_n followed by filter design formulas

n	g	ZoJn	Zoe	Zoo
1	1.6703	0.306664	70.04	39.37
2	1.1926	0.111295	56.18	45.05
3	2.3661	0.09351	55.11	45.76
4	0.8419	0.111294	56.18	45.05
5	1.9841	0.306653	70.03	39.37

ADS – coupled line BPF



ADS – coupled line BPF



Examples

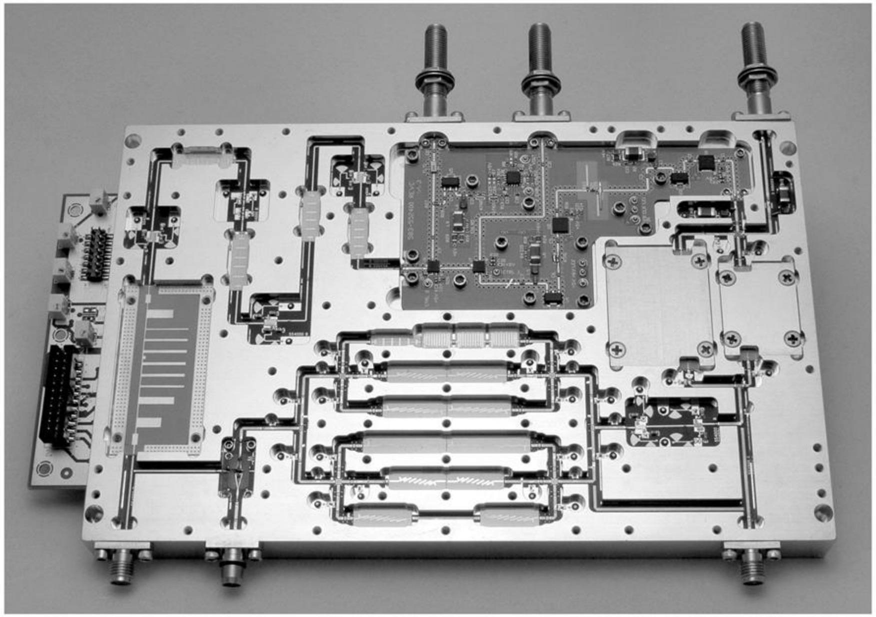
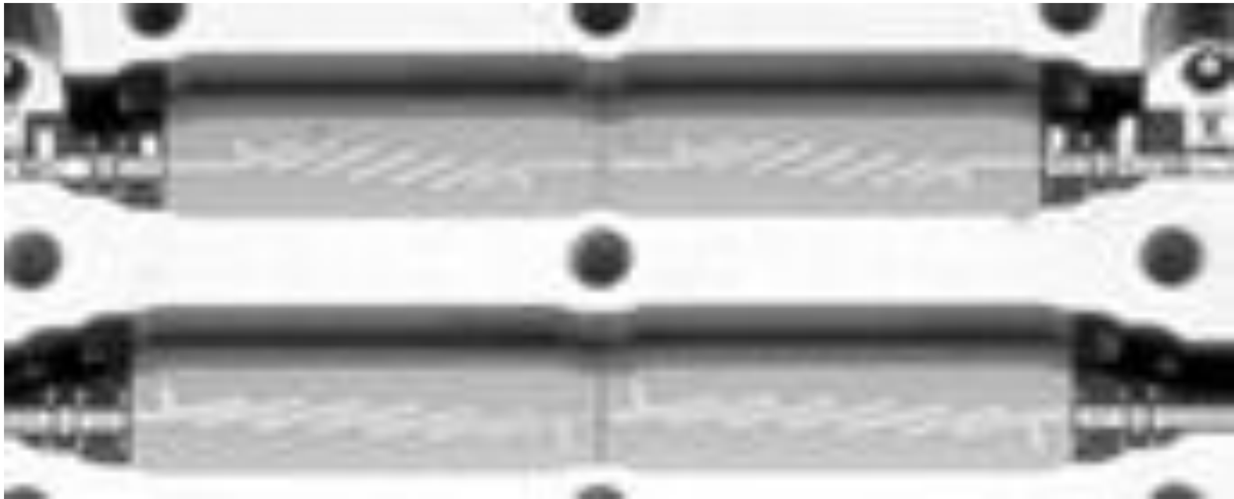
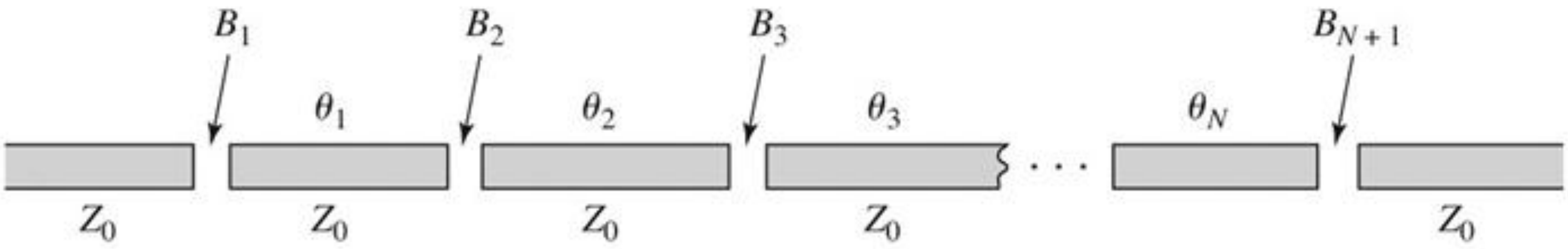


Figure 8.55
Courtesy of LNX Corporation, Salem, N.H.

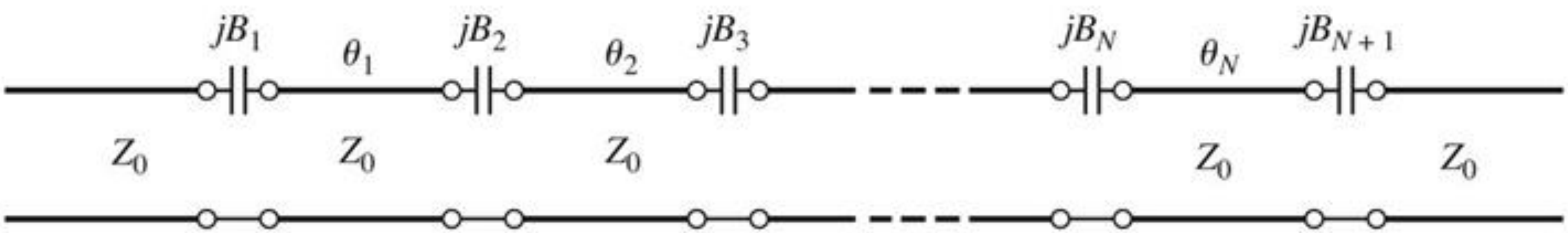


Bandpass Filters Using Capacitively Coupled Series Resonators

- The gaps between the resonators (~λ/2) generate a capacitive coupling between two resonators and can be approximated as series capacitors



(a)



(b)

Bandpass Filters Using Capacitively Coupled Series Resonators

- From the real physical length of the resonators, some part is used implement a admittance inverter (the remainder $\phi=\pi$, $l=\lambda/2$, resonator)

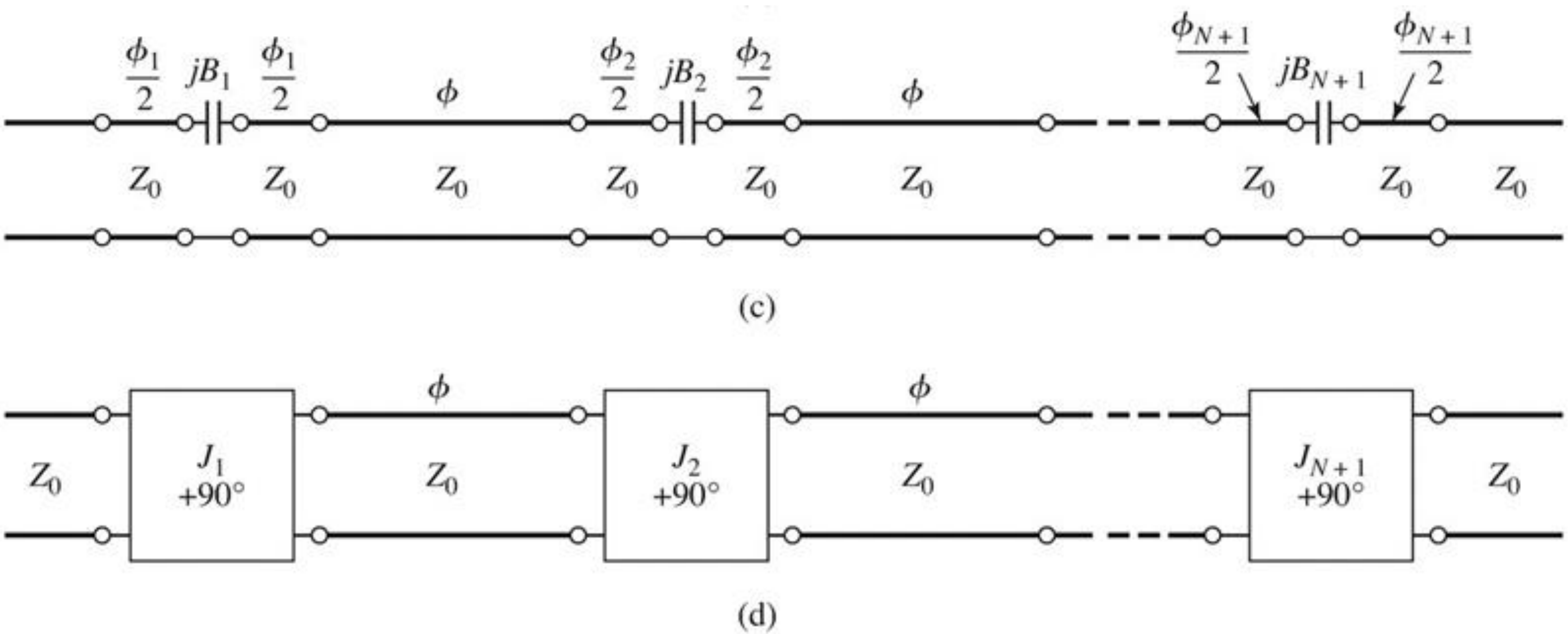


Figure 8.50

Bandpass Filters Using Capacitively Coupled Series Resonators design

- Compute the inverters (similar to coupled lines)

$$Z_0 \cdot J_1 = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_1}} \qquad Z_0 \cdot J_n = \frac{\pi \cdot \Delta}{2 \cdot \sqrt{g_{n-1} \cdot g_n}}, n = \overline{2, N} \qquad Z_0 \cdot J_{N+1} = \sqrt{\frac{\pi \cdot \Delta}{2 \cdot g_N \cdot g_{N+1}}}$$

- Compute capacitive susceptances

$$B_n = \frac{J_n}{1 - (Z_0 \cdot J_n)^2}, n = \overline{1, N+1}$$

- Compute the line lengths that must be “borrowed” to implement the inverters

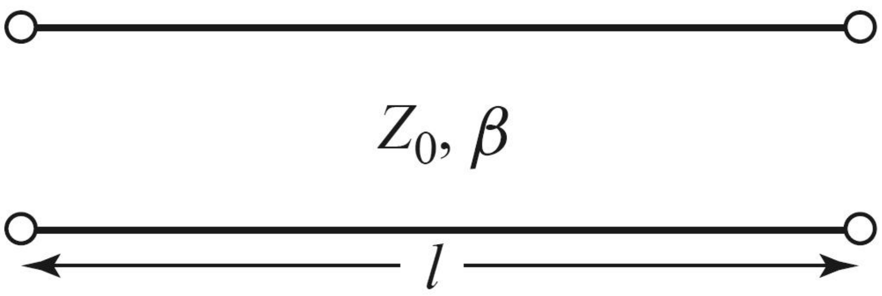
$$\phi_n = -\tan^{-1}(2 \cdot Z_0 \cdot B_n), n = \overline{1, N+1} \qquad \phi_n < 0, n = \overline{1, N+1}$$

- Compute the actual length of the lines (λ/2 + borr.)

$$\theta_i = \pi + \frac{1}{2} \cdot (\phi_i + \phi_{i+1}) = \pi - \frac{1}{2} \cdot \left[\tan^{-1}(2 \cdot Z_0 \cdot B_i) + \tan^{-1}(2 \cdot Z_0 \cdot B_{i+1})\right], i = \overline{1, N}$$

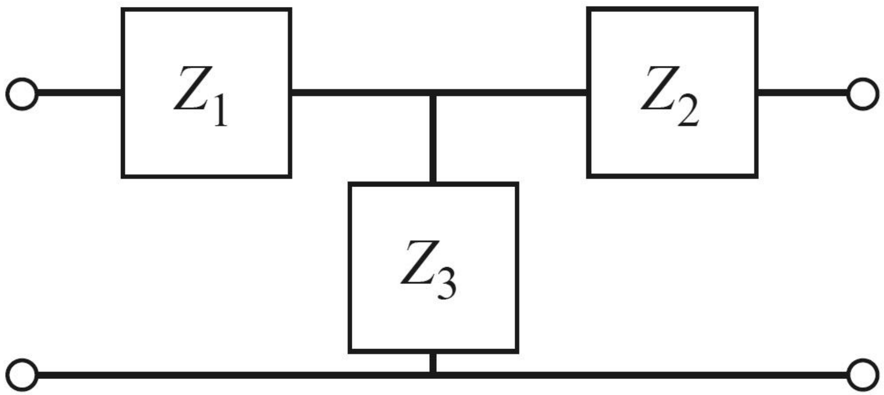
Equivalent circuits for short sections of transmission lines

- ABCD matrix (L4)
- short line , model with lumped elements is valid



$$A = \cos\beta \cdot l \qquad B = j \cdot Z_0 \cdot \sin\beta \cdot l$$

$$C = j \cdot Y_0 \cdot \sin\beta \cdot l \qquad D = \cos\beta \cdot l$$



$$A = 1 + \frac{Z_1}{Z_3} \qquad B = Z_1 + Z_2 + \frac{Z_1 \cdot Z_2}{Z_3}$$

$$C = \frac{1}{Z_3} \qquad D = 1 + \frac{Z_2}{Z_3}$$

Equivalent circuits for short sections of transmission lines

- The shunt element is capacitive

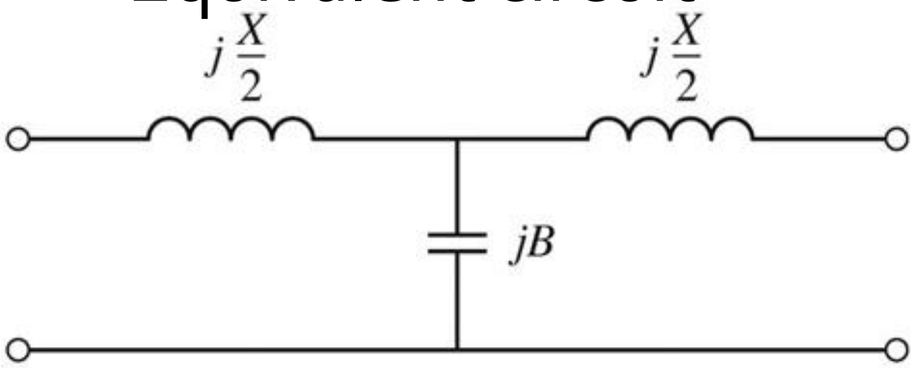
$$Z_3 = \frac{1}{j \cdot Y_0 \cdot \sin \beta \cdot l}$$

- Series elements are equal, and inductive

$$\cos \beta \cdot l = 1 + \frac{Z_1}{Z_3} = 1 + \frac{Z_2}{Z_3}$$

$$Z_1 = Z_2 = Z_3 \cdot (\cos \beta \cdot l - 1) = -j \cdot Z_0 \cdot \frac{\cos \beta \cdot l - 1}{\sin \beta \cdot l} = j \cdot Z_0 \cdot \tan \frac{\beta \cdot l}{2}$$

- Equivalent circuit

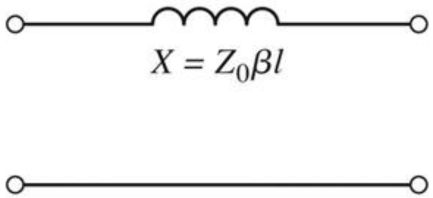


$$\frac{X}{2} = Z_0 \cdot \tan \frac{\beta \cdot l}{2}$$

$$B = \frac{1}{Z_0} \cdot \sin \beta \cdot l$$

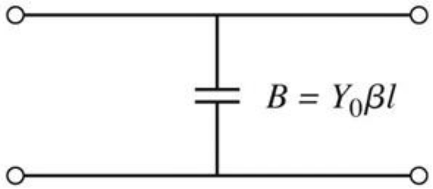
Equivalent circuits for short sections of transmission lines

- depending on the characteristic impedance:
  - high Zo >>



$$X \cong Z_0 \cdot \beta \cdot l \qquad \beta \cdot l < \frac{\pi}{4} \qquad Z_0 = Z_h$$

  - low Zo <<



$$B \cong Y_0 \cdot \beta \cdot l \qquad \beta \cdot l < \frac{\pi}{4} \qquad Z_0 = Z_l$$

Stepped-impedance low-pass filter

- Series L, shunt C, we realize low-pass filters
- We use
  - lines with high characteristic impedance to implement an series inductor

$$\beta \cdot l = \frac{L \cdot R_0}{Z_h}$$

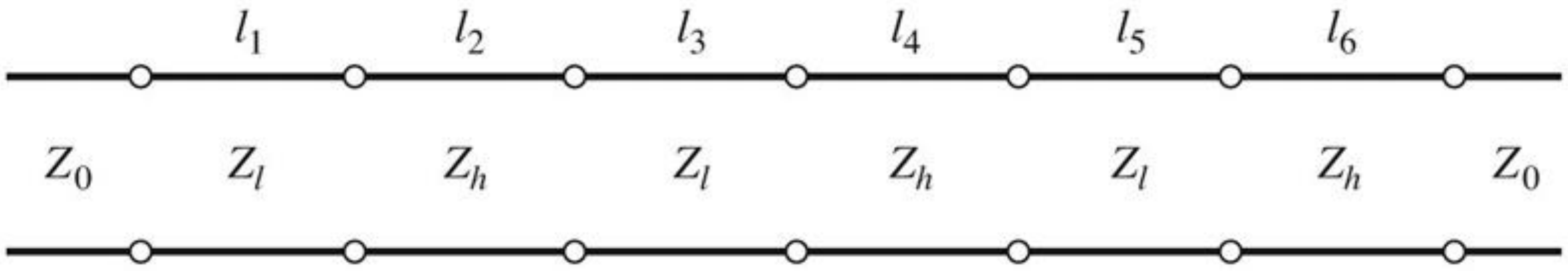
  - lines with low characteristic impedance to implement a shunt capacitor

$$\beta \cdot l = \frac{C \cdot Z_l}{R_0}$$

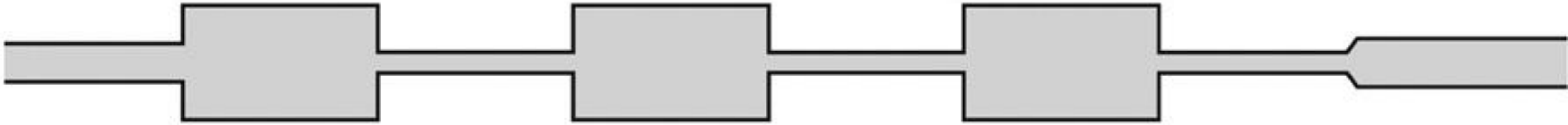
- usually the highest and lowest characteristic impedance that can be practically fabricated

Stepped-impedance LPF

- Not all the lines will result with the same length so the filter response is not periodic in frequency



(b)



(c)

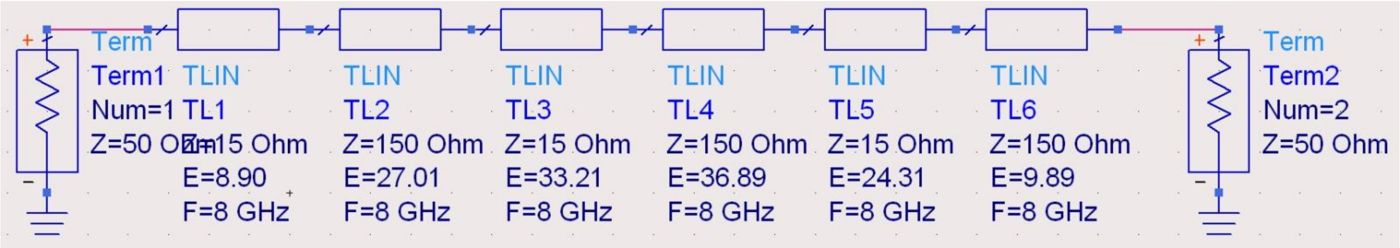
Figure 8.40

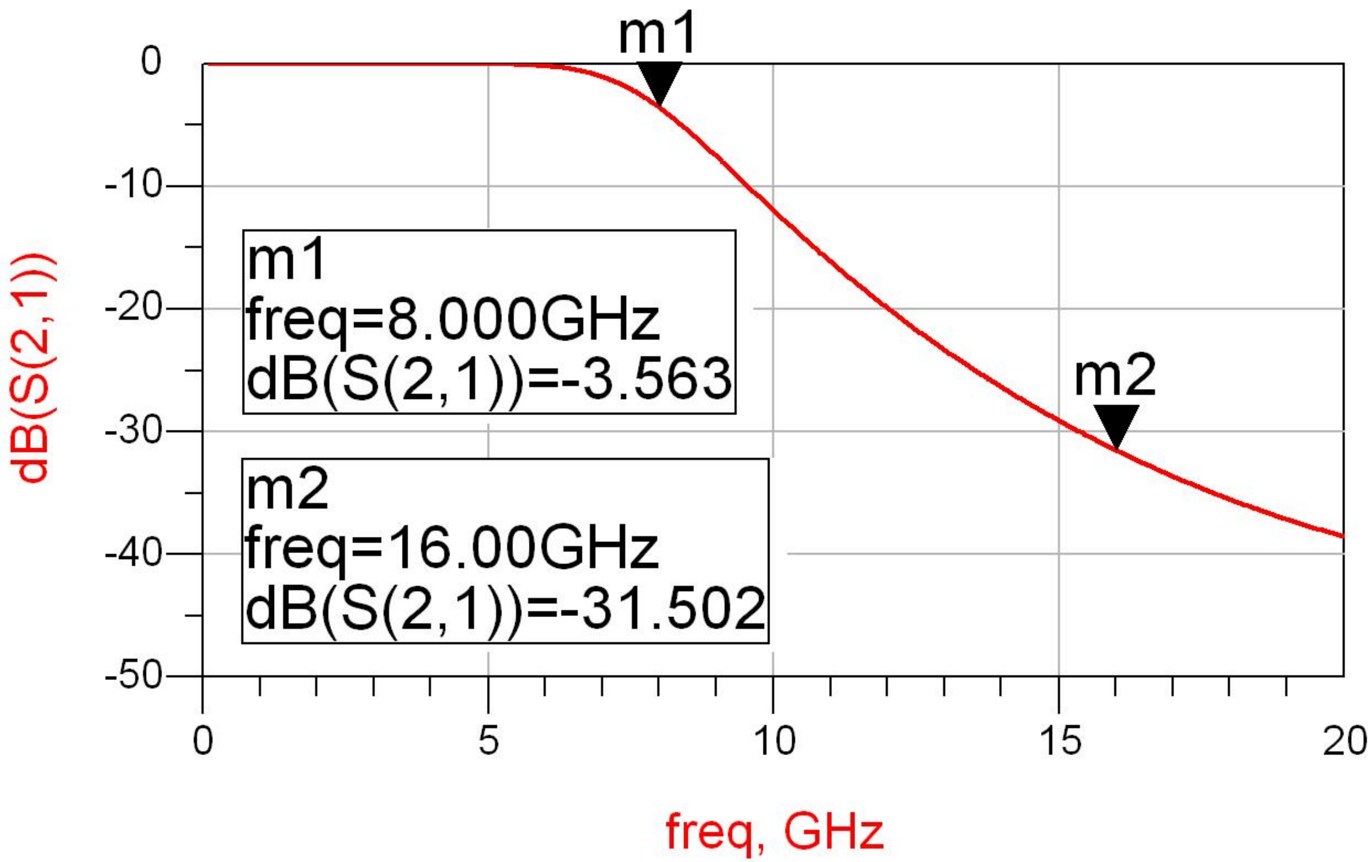
Example

- LPF with 8GHz cutoff frequency, 6^{th} order. Maximum realizable impedance is 150Ω and lowest 15Ω.

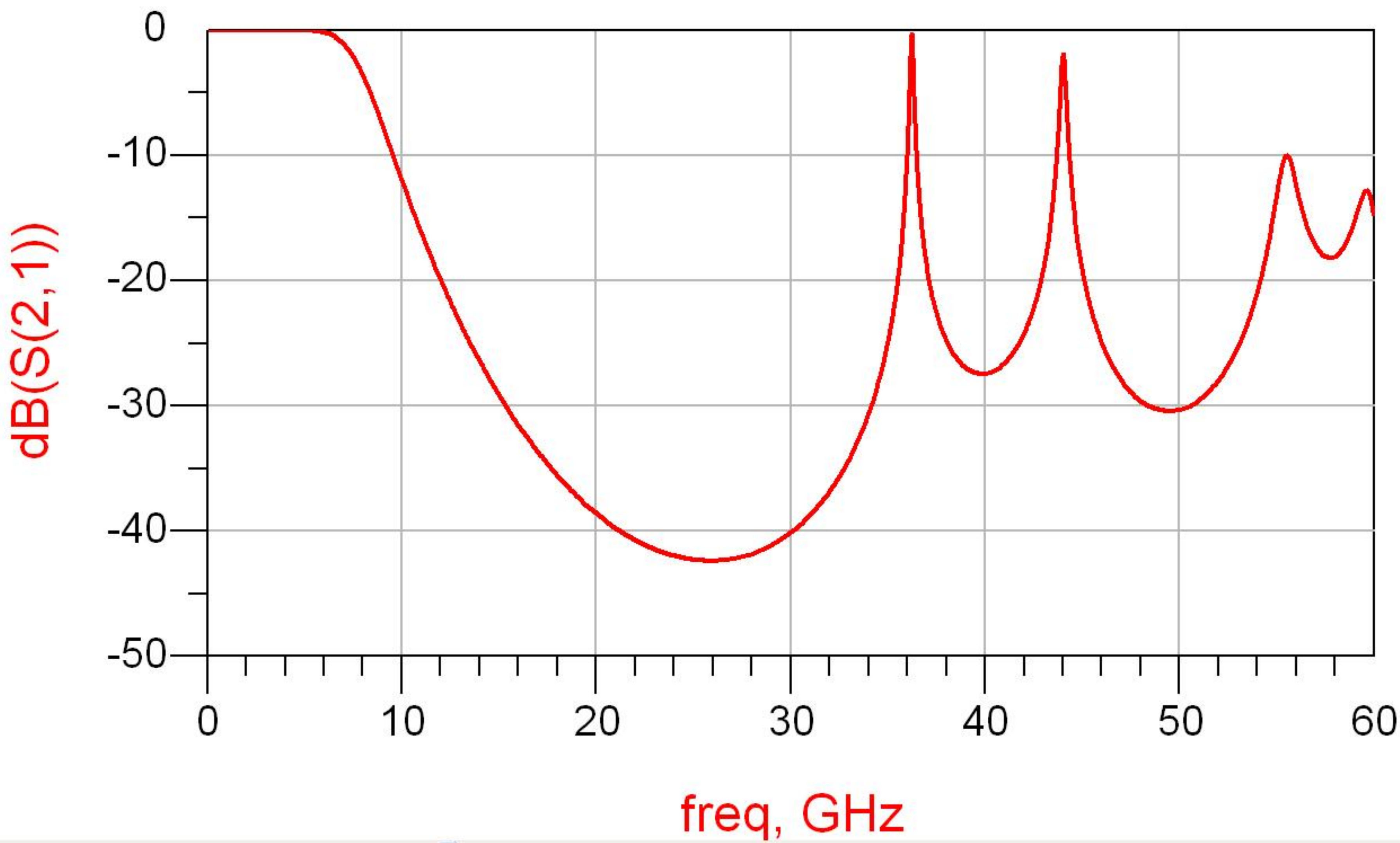
n	g_n	L/C_n	Z	θ_n[rad]	θ_n[°]
1	0.5176	0.206pF	15	0.155	8.90
2	1.4142	1.407nH	150	0.471	27.01
3	1.9318	0.769pF	15	0.580	33.21
4	1.9318	1.922nH	150	0.644	36.89
5	1.4142	0.563pF	15	0.424	24.31
6	0.5176	0.515nH	150	0.173	9.89

ADS – Stepped-impedance LPF

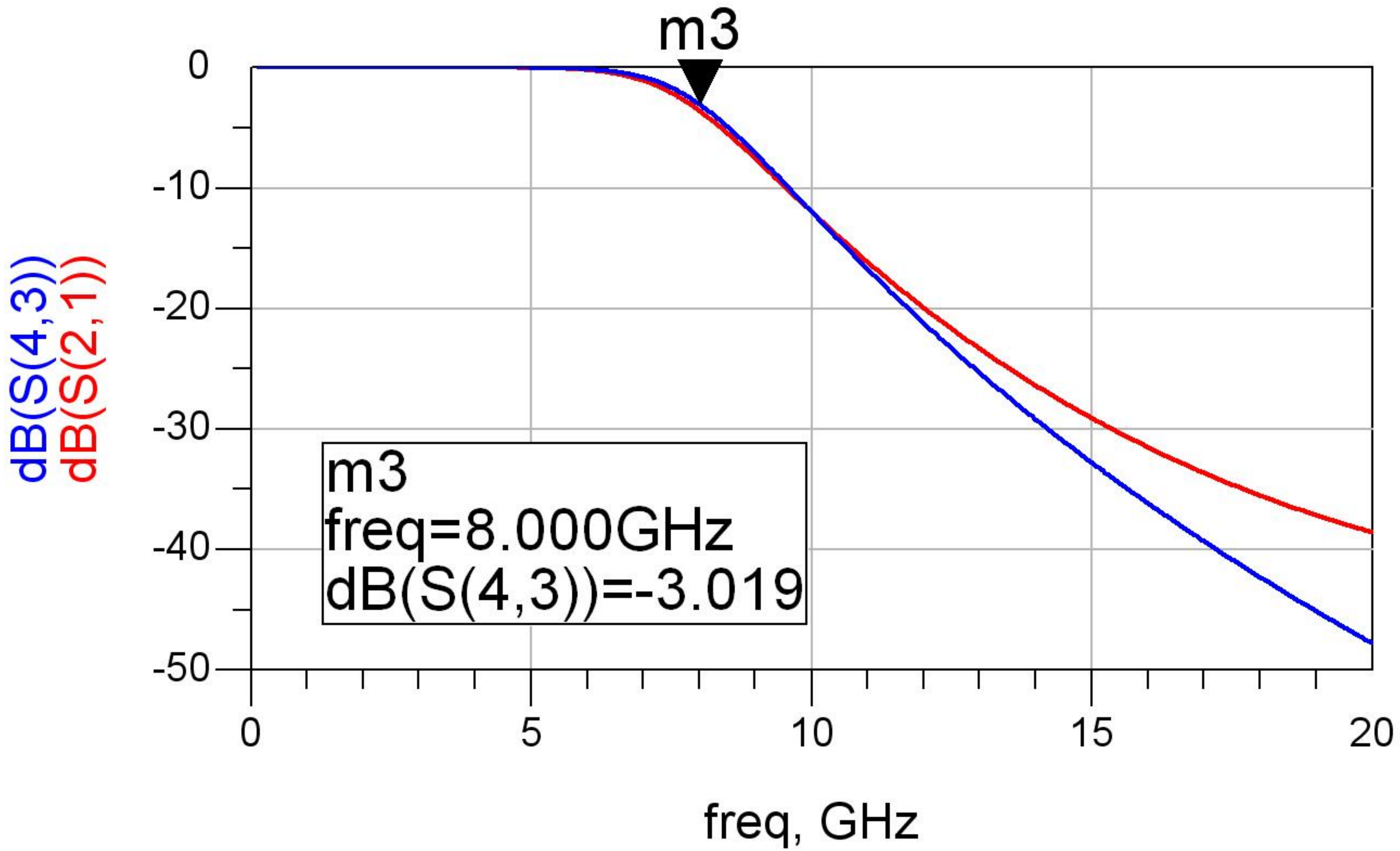




ADS – Stepped-impedance LPF



ADS – Stepped-impedance LPF – compared with lumped elements



Examples

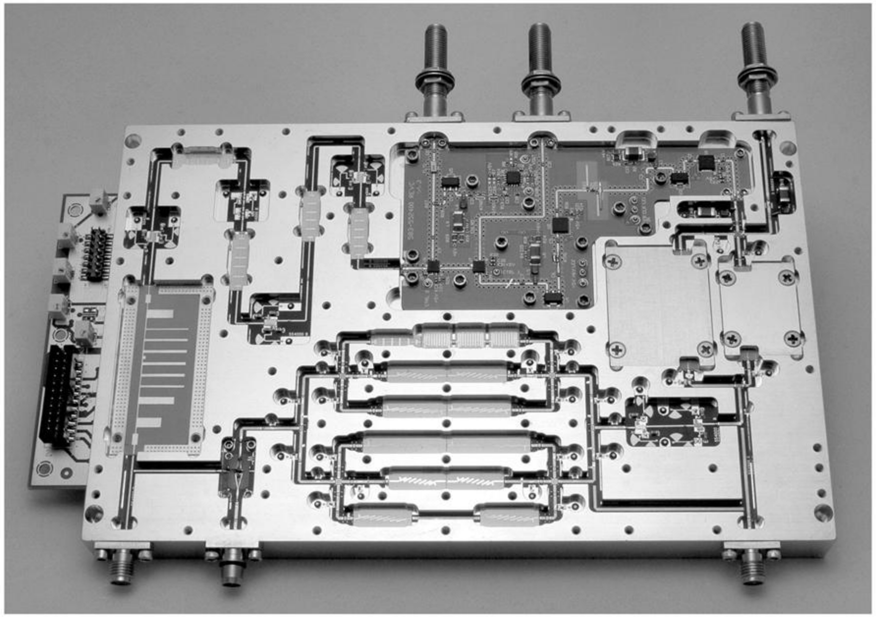
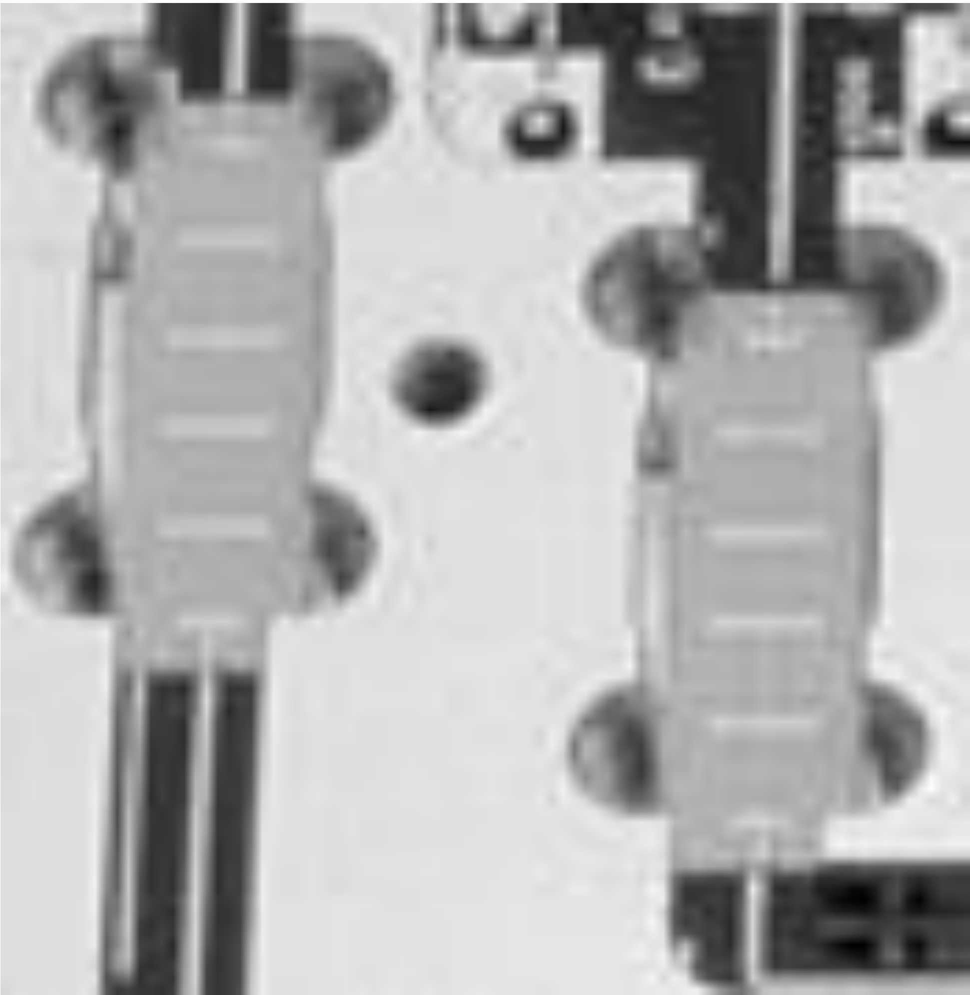


Figure 8.55
Courtesy of LNX Corporation, Salem, N.H.



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